E.Q.:

How do we use a quadratic model to represent a real world situation?

#2) $(x)(x-4)=32$

$x^2-4x=32$

$x^2-4x+\underline{4}=32+\underline{4}$

$(x-2)^2=36$

$x-2=\pm 6$

$x=2\pm 6$

8 or $\cancel{-4}$

#3) $(3x)(3x)=81$

$\frac{9x^2}{9}=\frac{81}{9}$

$x^2=9$

$x=\pm 3$

$x=3$

$\sqrt{x^2}=\sqrt{9}$

$x=3$

#4) $(3x+2)(x) = 16$

$3x^2 + 2x = 16$

$3x^2 + 2x - 16 = 0$

$$\begin{array}{ccc} & -48 & \\ \underline{8} & & \underline{-6} \\ & 2 & \end{array}$$

$3x^2 + 8x - 6x - 16 = 0$

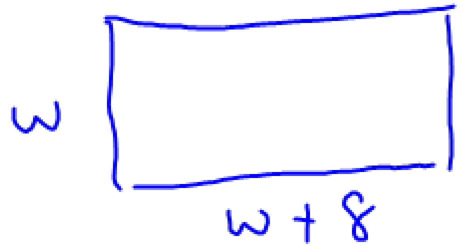
$x(3x+8) \quad -2(3x+8)$

$(x-2)(3x+8) = 0$

$x = 2 \qquad x = \cancel{-\frac{8}{3}}$

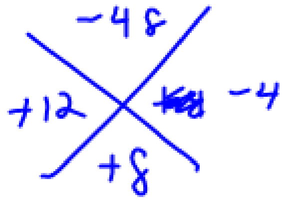
#5) Let w = width of the pad

Let $w+8$ = length of the pad



$$w(w+8) = 48$$

$$w^2 + 8w = 48$$

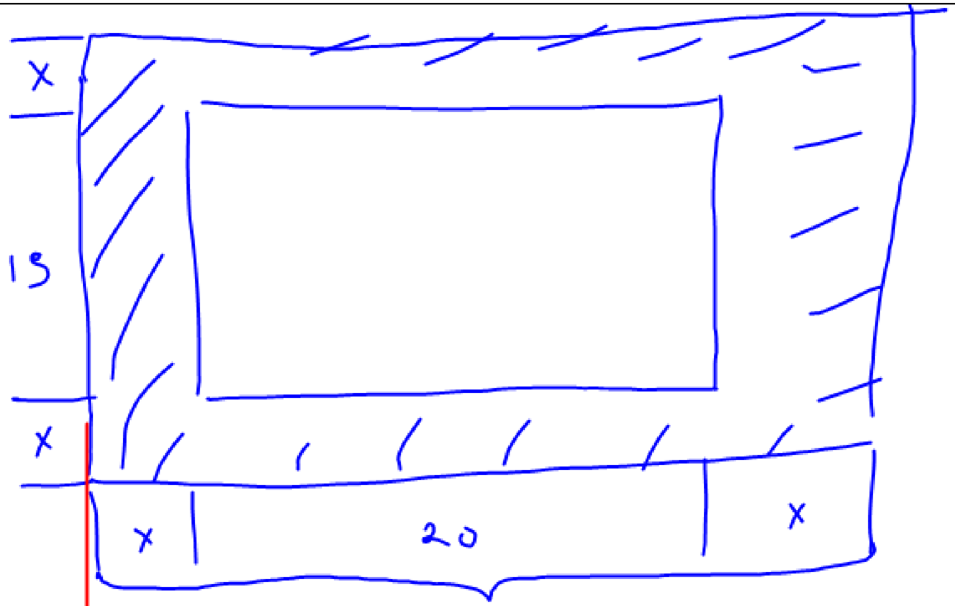


$$w^2 + 8w - 48 = 0$$

$$(w+12)(w-4)$$

$$\cancel{w = -12} \qquad w = 4$$

#6)



$x + 20 + x$

$$(2x + 20)(2x + 15) = 414$$

$$4x^2 + 30x + 40x + 300 = 414$$

$$4x^2 + 70x + 300 = 414$$

$$4x^2 + 70x - 114 = 0$$

$a = 4$

$b = 70$

$c = -114$

$$\frac{-70 \pm \sqrt{(70)^2 - 4(4)(-114)}}{2(4)} = \frac{-70 \pm \sqrt{6724}}{8}$$

$$\frac{-70 \pm 82}{8}$$

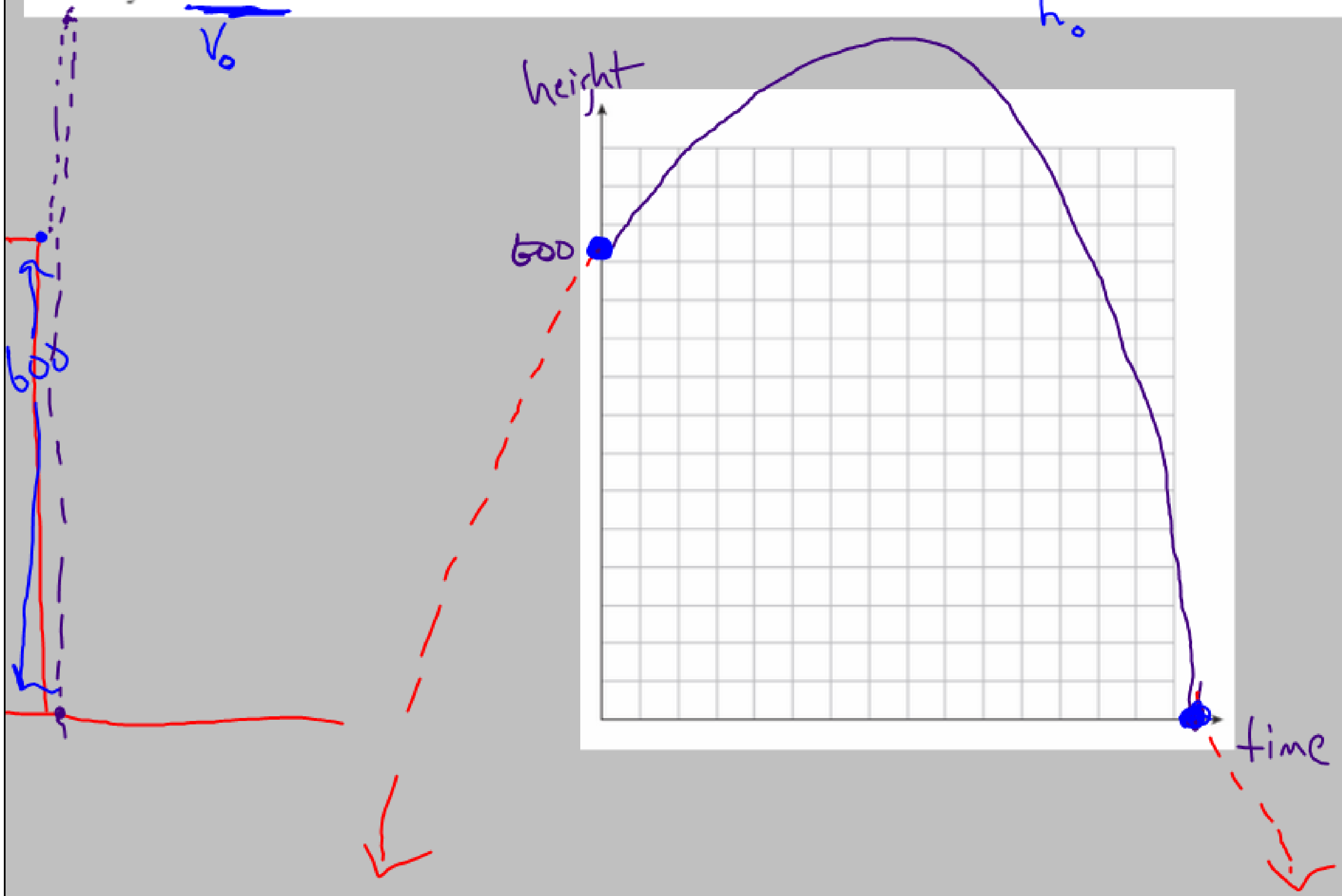
$$\frac{-70+82}{8} = \frac{3}{2} = 1.5 \text{ ft} \qquad \frac{-70-82}{8} = \cancel{-19}$$

Vertical Motion Problems

If an object is projected straight upward at time $t = 0$ from a point h_0 feet above ground, with an initial velocity v_0 ft/sec, then its height above ground after t seconds is given by $\underline{h(t)} = \underbrace{-16t^2 + v_0 t + h_0}$.

$$y = -16x^2 + v_0 x + h_0$$

Example 1: A projectile is fired vertically upward from a height of 600 feet above the ground, with an initial velocity of 803 ft/sec.



$$h(t) = -16t^2 + v_0 t + h_0$$

$$h_0 = initial\ height$$

$$v_0 = initial\ velocity$$

Example 1: A projectile is fired vertically upward from a height of 600 feet above the ground, with an initial velocity of 803 ft/sec.

(a) Write a quadratic model for its height $h(t)$ in feet above the ground after t seconds.

$$h(t) = -16t^2 + 803t + 600$$

$$h(t) = -16t^2 + 803t + 600$$

(b) How high is the projectile after 10 seconds?

time

height?

$$h(10) = -16(10)^2 + 803(10) + 600$$

$h(10) = 7{,}030$ feet

$$h(t) = -16t^2 + 803t + 600$$

(c) During what time interval will the projectile be more than 5000 feet above the ground?

time?

height

$$\begin{aligned} 5000 &= -16t^2 + 803t + 600 \\ -5000 & \qquad\qquad\qquad\qquad -5000 \\ 0 &= -16t^2 + 803t - 4400 \end{aligned}$$

$a = -16$

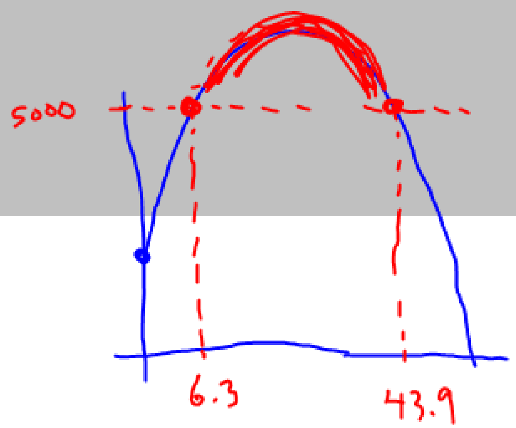
$b = 803$

$c = -4400$

$$\frac{-803 \pm \sqrt{(803)^2 - 4(-16)(-4400)}}{2(-16)}$$

$$\frac{-803 \pm 602.7}{-32} = \frac{-803 + 602.7}{-32} = 6.3$$

$$\frac{-803 - 602.7}{-32} = 43.9$$



$$\boxed{(6.3, 43.9)}$$

Domain $[0, 50.9]$

$$h(t) = -16t^2 + 803t + 600$$

(d) How long will the projectile be in flight?

50.9 sec

$0 = -16t^2 + 803t + 600$

$a = -16$

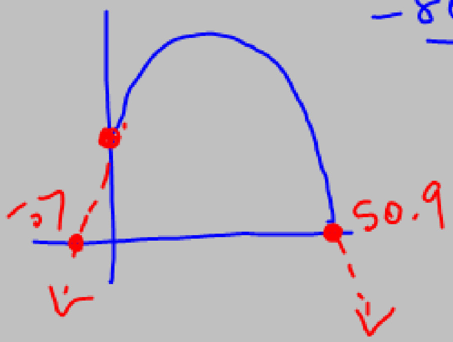
$b = 803$

$c = 600$

$$\frac{-803 \pm \sqrt{(803)^2 - 4(-16)(600)}}{2(-16)}$$

$$\frac{-803 \pm 826.6}{-32} = \frac{-803 + 826.6}{-32} = -.7$$

$$\frac{-803 - 826.6}{-32} = 50.9$$



Range $[0, 10675.1]$

$$h(t) = -16t^2 + 803t + 600$$

(e) What is the maximum height the projectile reaches?

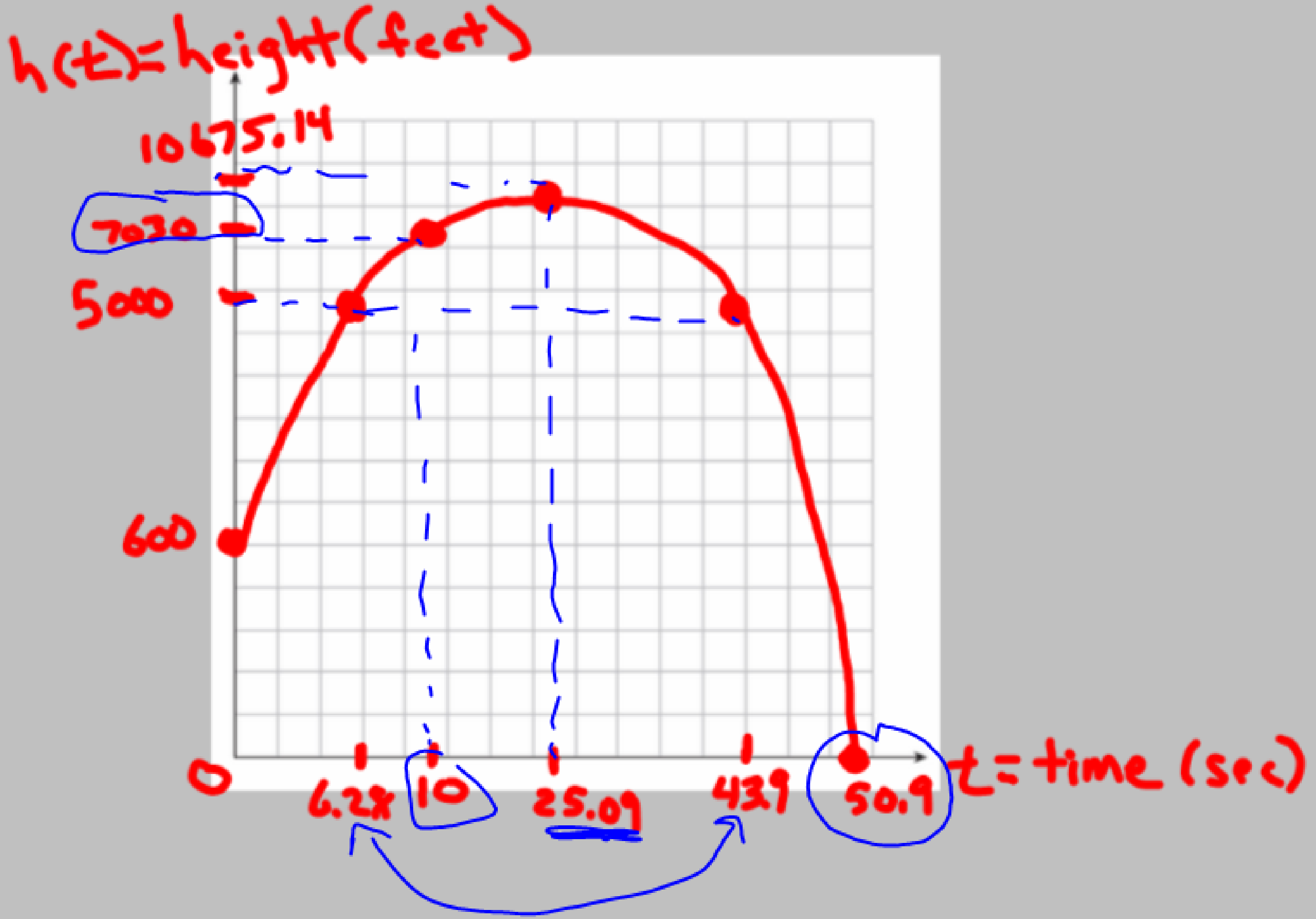
10,675.1 feet

$h(25.1) = -16(25.1)^2 + 803(25.1) + 600$

(f) At what time is the projectile this high?

25.1 sec

$$\frac{-b}{2a} = \frac{-803}{2(-16)} = \frac{-803}{-32} = 25.1$$


h(t)=height(feet)
10675.14
7030
5000
600
0
6.28
10
25.09
43.9
50.9
t=time (sec)

Example 2: An apple is thrown upward from 5 feet above the ground, with an initial velocity of 30 ft/sec.

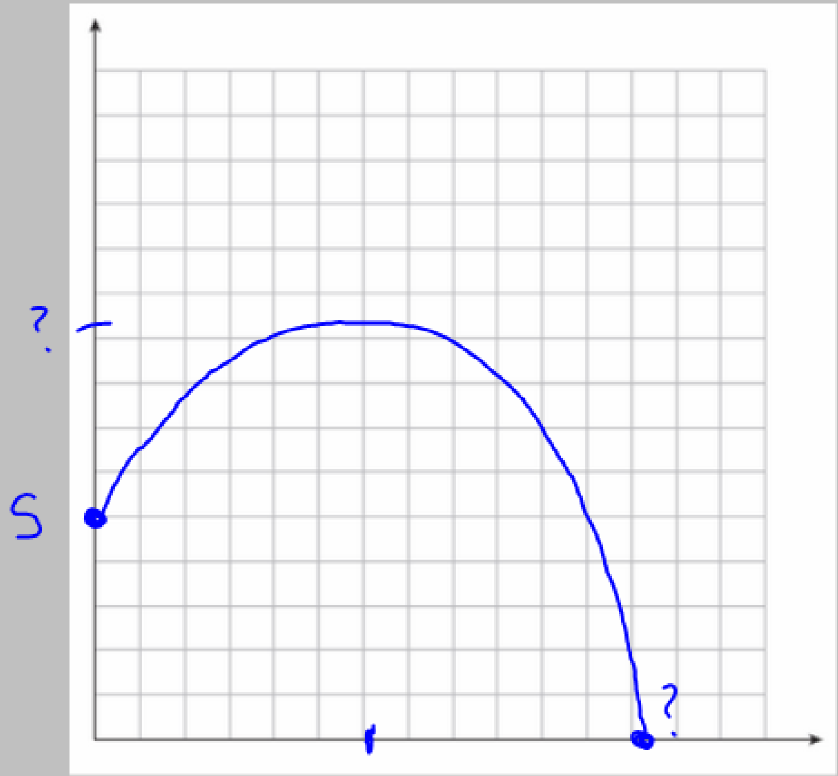
(a) Write a quadratic model for its height $h(t)$ in feet above the ground after t seconds.

$$h(t) = -16t^2 + v_0 t + h_0$$

$h_0 = initial\ height$

$v_0 = initial\ velocity$

$h(t) = -16t^2 + 30t + 5$



$$h(t) = -16t^2 + 30t + 5$$

height

(b) During what time interval will the apple be more than 15 feet above the ground?

(.4, 1.4)

$$15 = -16t^2 + 30t + 5$$

$$-15 \qquad\qquad -15$$

$$0 = -16t^2 + 30t - 10$$

h(t) = 15

find t

$$\frac{-30 \pm 16.1}{-32}$$

$$\frac{-30 + 16.1}{-32} \qquad .4$$

$$\frac{-30 - 16.1}{-32} \qquad 1.4$$

$$h(t) = -16t^2 + 30t + 5$$

(c) How long will the apple be in flight?

$$h(t) = -16t^2 + 30t + 5$$

(d) What is the maximum height the apple reaches? When does it get this high?

$-b$

Quadratic Population Models: $P(t) = P_0 + bt + at^2$

We refer to P_0 as the initial population.

Example 3: Suppose that the future population of Stockton City t years after January 1, 2000 is described (in thousands) by the quadratic model $P(t) = 110 + 4t + 0.07t^2$.

(a) What is the population of Stockton City on January 1, 2007?

(b) When will the population of Stockton City reach 180 thousand?

HW #13:

Quadratic Models