Warmup:

$$\frac{x^3 - 5x^2 - 2x + 24}{\boxed{x^1 - 3}} = (x^2 - 2x - 8)$$

linear

$$\begin{array}{r|l} & x^2 - 2x - 8 \\ \hline x - 3 & \cancel{x^3} - 5x^2 - 2x + 24 \\ & -\cancel{x^3} + 3x^2 \\ \hline & \cancel{-2x^2} - 2x \\ & \cancel{+2x^2} \mp 6x \\ \hline & -8\cancel{x} + 2\cancel{4} \\ & +\cancel{8}x \mp \cancel{24} \\ \hline & 0 \end{array}$$

$$\underbrace{x^3 - 5x^2 - 2x + 24} = (\underline{x-3}) \cdot (\underline{x^2 - 2x - 8})$$

Synthetic Division

- Synthetic division is a shorthand, or shortcut, method of polynomial division in the special case of dividing by a linear factor and it only works in this case.

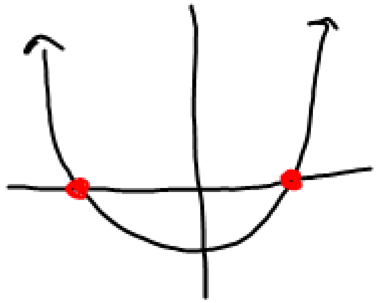
$$\frac{x^2-5x-3}{x+4}$$ linear ✓

$$\frac{x^3-5x+6}{3x}$$ linear ✓

$$\frac{x^4+6x-12}{x^2-4x+1}$$ Quadratic X

- Synthetic division is generally used, however, not for dividing out factors but for finding zeroes of polynomials.

zeroes or x-intercepts or roots



Synthetic division is often used to find the roots of higher-degree polynomials (degree 3 and up). These roots can be used to factor the polynomial.

Using Synthetic Division

So how does synthetic division work? Synthetic division is a shortcut to long division. It's a method that uses only the coefficients of the terms to save time.

Synthetic division can only be used if you're dividing by a LINEAR factor in the form x - a.

Division Problem:

$$\frac{x^3 - 5x^2 - 2x + 24}{x - 3}$$

Step 1: Make sure the terms of the numerator are in descending order. If a term is missing, add it in with a coefficient of 0.

Make sure the terms are in order from
highest degree to lowest degree.

$$x^3 - 5x^2 - 2x + 24$$

Step 2: Set the denominator equal to 0 and solve to find the number to put as the divisor. When you use long division, you subtract at each step. Synthetic division uses addition instead, so we switch the sign to account for this. If you're dividing by x - 4, you'll use a positive 4. If you're dividing by x + 5, you'll use a -5.

$$\begin{aligned} x - 3 &= 0 \\ +3 \quad &\;\; +3 \\ \hline x &= 3 \end{aligned}$$

Set the denominator equal to 0 and solve to find the number to use on the outside.

Step 3: Set up the problem using only the coefficients of each term in the numerator.

Set up the problem similar to a long division problem,
but using only the coefficients of each term.

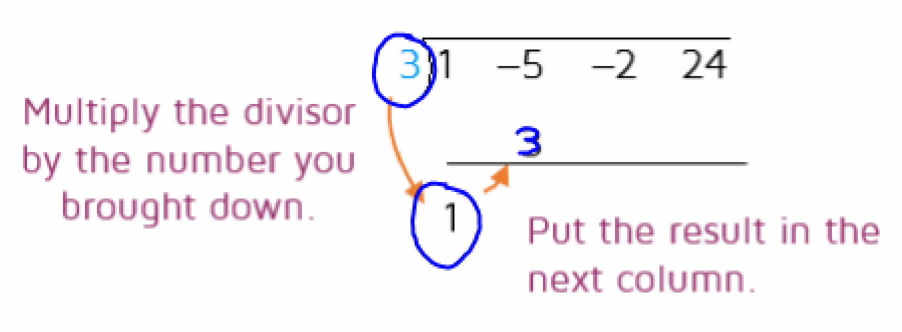
$$3\,\overline{)\,1 \quad -5 \quad -2 \quad 24}$$

Step 4: Bring down the first coefficient. When you're dividing by x - a, the first coefficient will always stay the same.

$$\begin{array}{r|rrrr} 3 & 1 & -5 & -2 & 24 \\ & \downarrow & & & \\ \hline & 1 & & & \end{array}$$

Bring down the
first coefficient.

Step 5: Multiply the divisor by the number you brought down. Put the result in the next column.



Step 6: Add the numbers in the 2nd column. In long division, you subtract. With synthetic division, we switch the sign when we write the divisor so that we can **add**.

$$
\begin{array}{r|rrrr}
3 & 1 & -5 & -2 & 24 \\
 & & 3 & & \\
\hline
 & 1 & -2 & &
\end{array}
$$

Add the numbers in the 2^{nd} column.

Step 7: Repeat. Multiply the divisor by the new number you wrote down and put the result in the next column. Repeat this process until you run out of columns. The last number you write down is the remainder.

Multiply the divisor by the next number.

$$\begin{array}{r|rrrr} 3 & 1 & -5 & -2 & 24 \\ & & 3 & -6 & \\ \hline & 1 & -2 & -8 & \end{array}$$

Add the numbers in the 3rd column.

Repeat.

$$\begin{array}{r|rrrr} 3 & 1 & -5 & -2 & 24 \\ & & 3 & -6 & -24 \\ \hline & 1 & -2 & -8 & \boxed{0} \end{array}$$

The last number you write down is the remainder.

Step 8: Write the answer. The numbers you wrote down on the bottom row are the **coefficients** of the answer. The last number on the right is the remainder. In synthetic division, you're always dividing by a linear factor in the form x - a, so the degree of your answer will always be one less than what you started with. For example, if the numerator had degree 4, then the answer would be degree 3. In this example, the numerator had degree 3, so our answer is a degree 2.

$$\begin{array}{r|rrrr} 3 & 1 & -5 & -2 & 24 \\ & & 3 & -6 & -24 \\ \hline & \boxed{1 \quad -2 \quad -8} & & & 0 \end{array}$$

0 Remainder

Coefficients of answer

$$\frac{x^3 - 5x^2 - 2x + 24}{x - 3}$$

Answer: $x^2 - 2x - 8$

Let's take a look at the two methods side by side. Synthetic division certainly takes up less space and the more you do it, the faster you'll get. Both methods give the same answer.

✓

Synthetic Division

$$\begin{array}{r|rrrr} 3 & 1 & -5 & -2 & 24 \\ & & 3 & -6 & -24 \\ \hline & 1 & -2 & -8 & 0 \end{array}$$

Answer: $x^2 - 2x - 8$

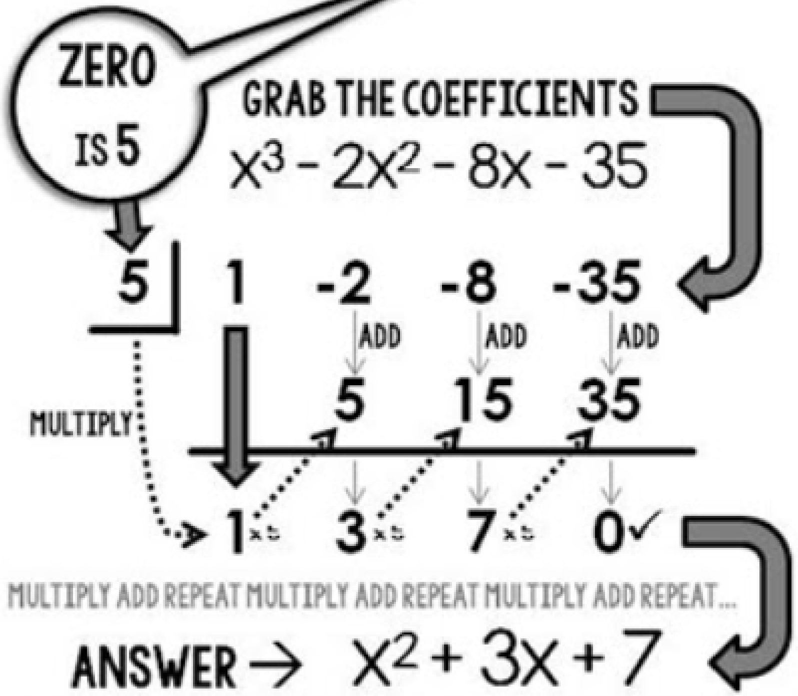
Long Division

$$\begin{array}{r l} & x^2 - 2x - 8 \\ x - 3 \,\big) & x^3 - 5x^2 - 2x + 24 \\ & \underline{x^3 - 3x^2} \\ & \quad -2x^2 - 2x + 24 \\ & \quad \underline{-2x^2 + 6x} \\ & \qquad\quad -8x + 24 \\ & \qquad\quad \underline{-8x + 24} \\ & \qquad\qquad\quad 0 \end{array}$$

SYNTHETIC DIVISION

EXAMPLE:

Divide:
$x^3 - 2x^2 - 8x - 35$
by $(x - 5)$.



STEPS

1: write the known zero in the house
2: list out the coefficients
3: bring down the 1st coefficient
4: multiply the 1st coefficient by house number
5: write the product under the 2nd coefficient
6: add down
7: repeat
8: use final numbers to write polynomial
9: use the Quadratic Formula to find the other zeros

$$\underbrace{(x^2 - 3x - 54)}_{\text{standard form}} \div \underbrace{(x - 9)}_{\text{linear}}$$

find the zero:

$$x - 9 = 0$$

$$x = 9$$

$$x^2 - 3x - 54 = (x-9)(x+6)$$

$$\begin{array}{r|rrr} 9 & 1 & -3 & -54 \\ & & 9 & 54 \\ \hline & 1 & 6 & \boxed{0} \end{array}$$

remainder

$$1x + 6$$

$(3x^2 + 20x - 16) \div (x + 8)$

$$
\begin{array}{r|rrr}
-8 & 3 & 20 & -16 \\
 & \downarrow & -24 & 32 \\
\hline
 & 3 & -4 & \boxed{16}
\end{array}
$$

$$3x - 4 + \frac{16}{x+8}$$

$(x^3 + 2x^2 - x + 6) \div (x + 4)$

-4	1	2	-1	6
	↓	-4	8	-28
	1	-2	7	-22

$x^2 - 2x + 7 \; \frac{-22}{x+4}$

$$\frac{x^3 - 1}{x - 1} = \frac{x^3 + 0x^2 + 0x - 1}{x - 1}$$

$$\begin{array}{r|rrrr}
1 & 1 & 0 & 0 & -1 \\
  & \downarrow & 1 & 1 & 1 \\
\hline
  & 1 & 1 & 1 & \boxed{0}
\end{array}$$

No remainder

$x^2 + x + 1$

$(3x^2 - 144) \div (x - 12)$

$$\begin{array}{r|rrr} 12 & 3 & 0 & -144 \\ & \downarrow & 36 & 432 \\ \hline & 3 & 36 & \boxed{288} \end{array}$$

$$3x + 36 + \frac{288}{x-12}$$

$(8x^2 + x) \div (x + 2)$

$$
\begin{array}{r|rrr}
-2 & 8 & 1 & 0 \\
 & \downarrow & -16 & 30 \\
\hline
 & 8 & -15 & \boxed{30}
\end{array}
$$

$$8x - 15 + \frac{30}{x+2}$$

$(4x^3 + x - 5) \div (x - 1)$

$$
\begin{array}{r|rrrr}
1 & 4 & 0 & 1 & -5 \\
 & \cancel{\downarrow} & 4 & 4 & 5 \\
\hline
 & 4 & 4 & 5 & \boxed{0}
\end{array}
$$

$4x^2 + 4x + 5$

HW #6:
Synthetic Division