Graphing Rational Functions

Rational Function: A function that is the quotient of two functions

$$f(x) = \frac{N(x)}{D(x)}$$

where N(x) and D(x) are polynomial functions

Parent Function

The parent function of a rational function is:

$x \neq 0$

$$f(x) = \frac{1}{x}$$

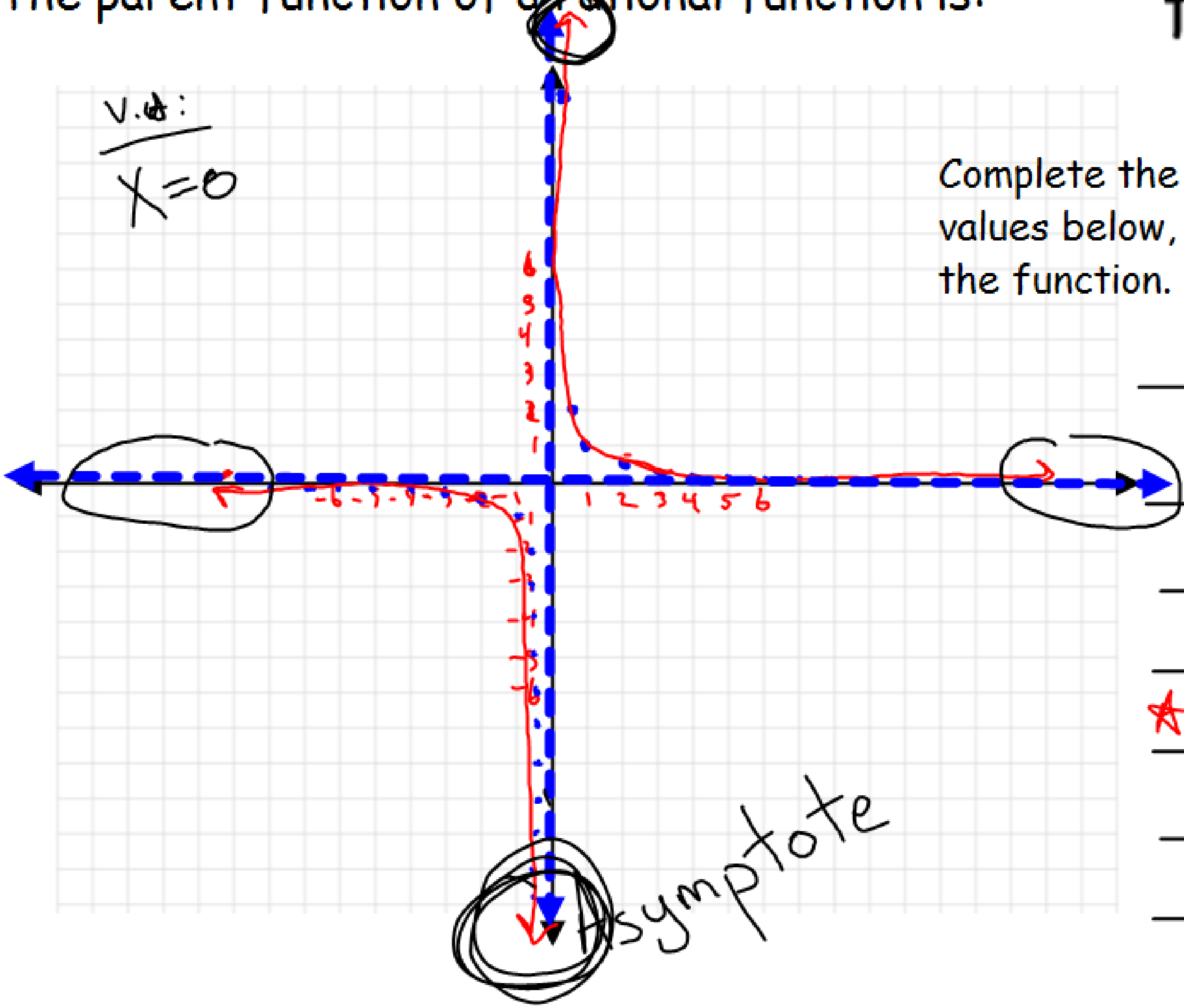
$\frac{1}{-3}$ $\frac{1}{-2}$ $\frac{1}{-1}$

$\frac{1}{1}$ $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$

$\frac{1}{.5} = 2$ $\frac{1}{.1}$

V.A.:

$x = 0$



Complete the table of values below, then graph the function.

x	y
-3	-.3
-2	-.5
-1	-1
★ 0	Undefined ★
1	1
2	.5
3	.3

Vocabulary:

Asymptote: A line that a curve approaches, as it heads towards infinity.

almost always

Vertical Asymptote: A line, which corresponds to zeros of the denominator of a rational function, that the graph will approach but never touch.

Usually V.A. & also H.A./SA

one or the other

Horizontal Asymptote: A line that the graph will approach as x gets infinitely bigger or smaller. A graph can and often does touch a Horizontal Asymptote.

Slant/Oblique Asymptote: When a Rational function does not have a Horizontal Asymptote, it will have an Oblique or slant asymptote instead.

Important Characteristics of Rational Functions:

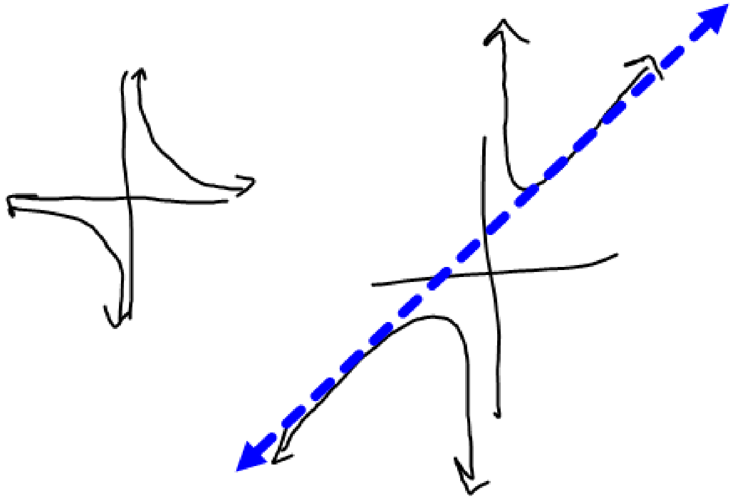
1. They have values at which the function is undefined (excluded values)

2. Those values either lead to the graph having a Vertical Asymptote or a Hole

3. The value of a Rational function always approaches a specific value or positive infinity or negative infinity

4. If the value approaches a specific value, the function would have a Horizontal Asymptote

5. If the value approaches infinity, the graph would have a Slant Asymptote instead of a Horizontal Asymptote



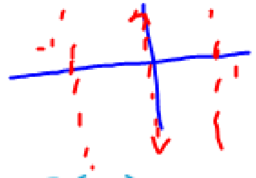
Finding Vertical Asymptotes of a Rational Function

- **Vertical Asymptotes** are vertical lines through the zeros of the denominator, or undefined values.
- The graph of a Rational Function will never intersect it's vertical asymptote
- There can be 0, 1 or more vertical asymptotes of a Rational Function

To find the V.A.'s:

Simply find the excluded values of the function

- set the denominator equal to zero and solve the resulting equation
- the V.A. is the line x = undefined value



Example 1: State the V.A. of f(x).

a) $f(x) = \dfrac{2x+1}{x^2-9}$

$x^2 - 9 = 0$

$\sqrt{x^2} = \sqrt{9}$

$x = 3$

$x = -3$

b) $f(x) = \dfrac{1}{x^3-x}$

V.A.

$x^3 - x = 0$

$(x)(x^2-1) = 0$

$x = 0$

$x^2 - 1 = 0$

$\sqrt{x^2} = \sqrt{1}$

$x = -1$

$x = 1$

c) $f(x) = \dfrac{x^2-7x+3}{x}$

V.A.

$x = 0$

Finding the Horizontal Asymptotes of Rational Functions

- The **Horizontal Asymptote** depends on the degree of both the numerator and the denominator
- There can be 0, or 1 Horizontal Asymptote of a Rational Function

$\rightarrow 4x^{3} + 3x^2 + 1$

$\rightarrow 3x^{2} + 4x - 1$

$\rightarrow 4x^{1} + 1$

5

To find the Horizontal Asymptote:

Find the degree of both the numerator,

top N(x) and denominator, D(x) bottom

- if the degree of N(x) < the degree of D(x), the H.A. is y = 0 (x-axis)
- if the degree of N(x) > the degree of D(x), there is no H.A. (Slant Asymptote)
- if the degree of N(x) = the degree of D(x), the H.A. is the ratio of the leading coefficients

Example 2: State the HA of f(x).

a) $f(x) = \frac{2x^2+5x}{2x^3+4}$

degree of top = 2

degree of bottom = 3

H.A: $y = 0$

b) $f(x) = \frac{9x^2+4x}{3x^2+1}$

top: 2

bottom = 2

HA: $y = \frac{9}{3}$

$y = 3$

c) $f(x) = \frac{7x^4+4x^2}{2x^2-5}$

top: 4

bottom: 2

No H.A.

Finding Holes of a Rational Function

Hole: Point of discontinuity. A point on a graph at which a function is not continuous.

$$\frac{x+2}{(x+1)(x+3)}$$

V.A. $x=-1$
$x=-3$

Finding the Holes (if any):

- Factor and simplify.
- Identify the zeros of N(x) and D(x)
- If any of the zeros match, the function will have a hole at that x value instead of a Vertical Asymptote.
- Plug x into the reduced function to solve for y
- The hole is the point (x,y)
- If graphing, put an open circle at that point.
- The graph of the function will touch that hole on both sides.

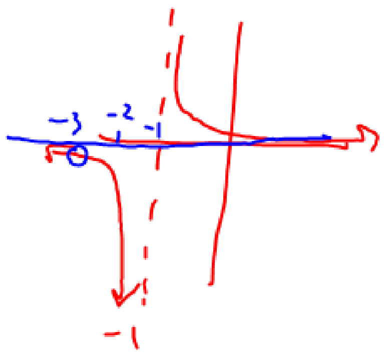
$$\frac{\cancel{x+3}^{\,1}}{(x+1)\cancel{(x+3)}}$$

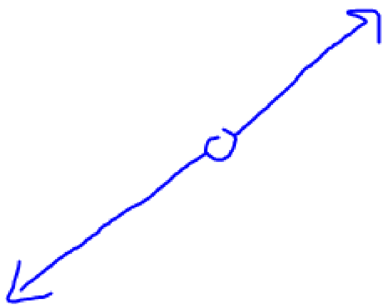
V.A: $x=-1$

Hole: $x=\underline{-3}$

$(-3, -.5)$

$$\frac{1}{-2} = \frac{1}{-3+1} = \frac{1}{x+1}$$





Example 3: Find the hole(s) of the rational function.

-3, +2, -6, -1

-3, 1, -3, -2

a) $f(x) = \frac{x^2-x-6}{x^2-2x-3} = \frac{\cancel{(x-3)}(x+2)}{\cancel{(x-3)}(x+1)} = \frac{x+2}{x+1}$

$\frac{3+2}{3+1} = \frac{5}{4}$

Hole

$\left(3, \frac{5}{4}\right)$

Hole

b) $f(x) = \frac{x^2+2x-24}{x^2-36} = \frac{\cancel{(x+6)}(x-4)}{\cancel{(x+6)}(x-6)} = \frac{x-4}{x-6}$

$= \frac{-6-4}{-6-6}$

$\frac{5}{6} = \frac{-10}{-12}$

$x+6=0$

$x=-6$

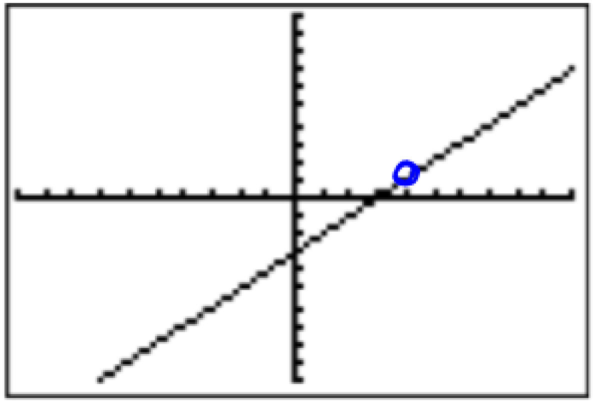
$(-6, 5/6)$

Example 4: Find the hole(s) of the rational functions and draw it on the graph.

a) $f(x) = \frac{x^2-7x+12}{x-4} = \frac{\cancel{(x-4)}(x-3)}{\cancel{(x-4)}} = \frac{x-3}{1}$

hole

(4, 1)

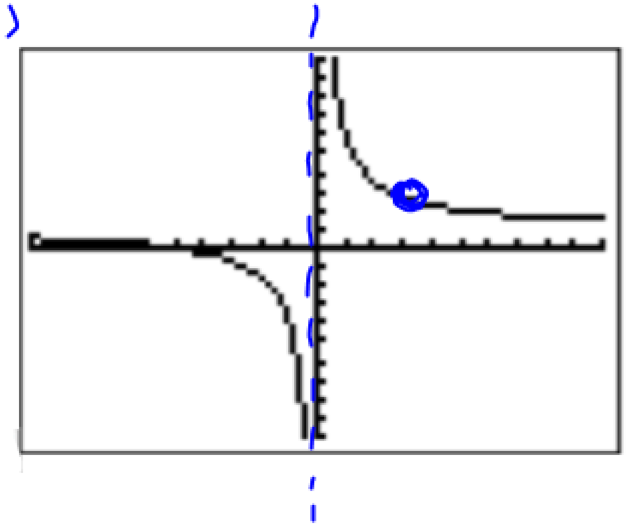


b) $f(x) = \frac{x^2+2x-15}{x^2-3x} = \frac{(x+5)\cancel{(x-3)}}{x\cancel{(x-3)}} = \frac{x+5}{x}$

hole

$\left(3, \frac{8}{3}\right)$

$(3, 2.\bar{6})$



HOMEWORK

VA/HA/HOLES WS