Find the roots (zeroes or x-intercepts) of the following functions:

Linear : Degree is 1

$y = 5x + 10$

$0 = 5x + 10$

$-10 \qquad -10$

$\frac{-10}{5} = \frac{5x}{5}$

$-2 = x$

$y = -3x - 12$

$0 = -3x - 12$

$+12 \qquad +12$

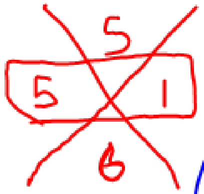
$\frac{12}{-3} = \frac{-3x}{-3}$

$-4 = x$

Find the roots (zeroes or x-intercepts) of the following functions:

Quadratic
Degree is 2

$y = x^2 + 6x + 5$

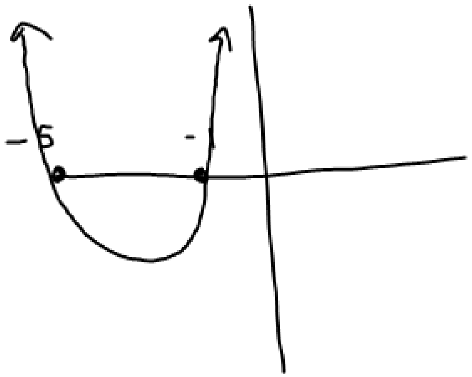


$y = (x+5)(x+1)$

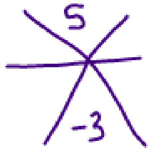
$0 = (x+5)(x+1)$

$0 = x+5$ $0 = x+1$

$x = -5$ $x = -1$



Solve Quadratics
- factoring
- quadratic formula
- completing the square
- Square root method



$y = x^2 - 3x + 5$

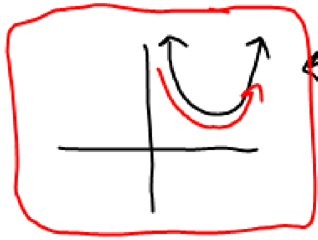
$a = 1$
$b = -3$
$c = 5$

$$\frac{(-b) \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{3 \pm \sqrt{(-3)^2 - 4(1)(5)}}{2(1)}$$

$$\frac{3 \pm \sqrt{9 - 20}}{2}$$

$$\frac{3 \pm \sqrt{-11}}{2} = \frac{3 \pm i\sqrt{11}}{2}$$



Fundamental Theorem of Algebra

The "Fundamental Theorem of Algebra" is **not** the start of algebra or anything, but it does say something interesting about polynomials :

Any polynomial of degree **n** has **n** roots
but we may need to use complex numbers
(imaginary #'s)

- A Polynomial looks like this:

$$2x^{4} + 6x - 5 = 0$$

terms

example of a polynomial
this one has 3 terms

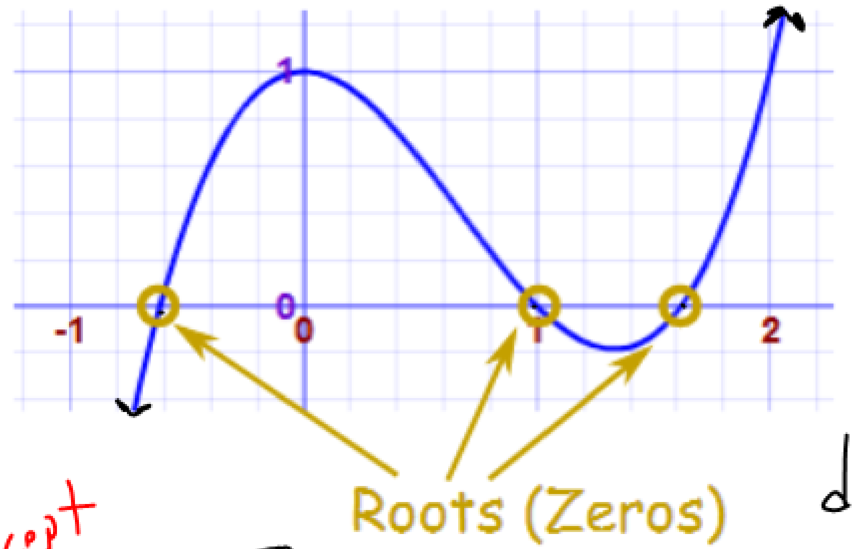
- The Degree of a Polynomial with one variable is ...

  ... the largest exponent of that variable.

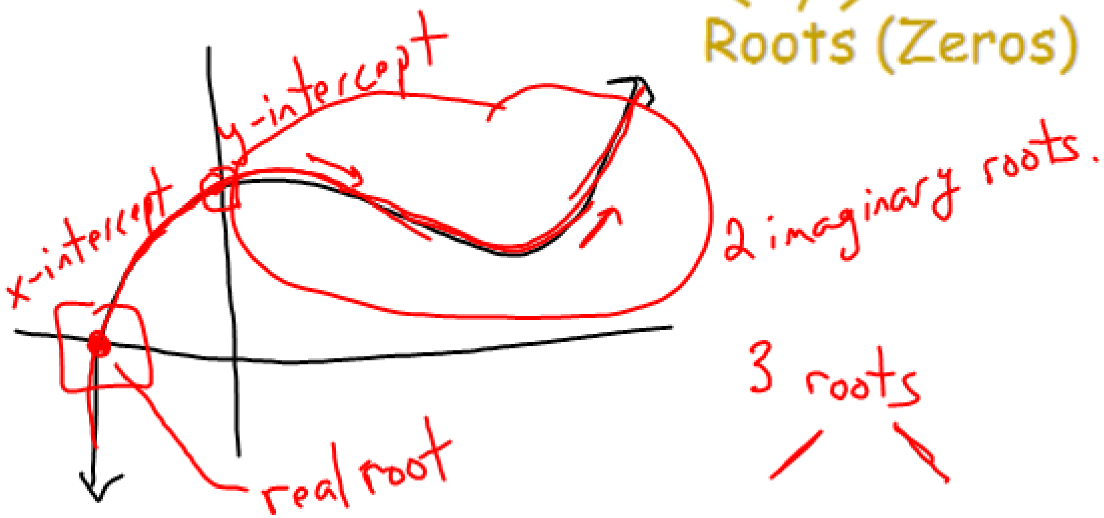
this makes it Degree 3

$$4x^3 + 2x^2 - 7$$

- A "root" (or "zero") is where the **polynomial is equal to zero**.



degree is 3



So, a polynomial of degree 3 will have 3 roots (places where the polynomial is equal to zero). A polynomial of degree 4 will have 4 roots. And so on.

The "Fundamental Theorem of Algebra" is **not** the start of algebra or anything, but it does say something interesting about polynomials :

Any polynomial of degree **n** has **n** roots
but we may need to use complex numbers

Example: what are the roots of $x^2 - 9$?

$x^2 - 9$ has a degree of 2 (the largest exponent of x is 2), so there are 2 roots.

Let us solve it. We want it to be equal to zero:

$$x^2 - 9 = 0$$

$(x+3)(x-3) = 0$

$x+3=0$ $\quad$ $x-3=0$

$x=-3$ $\quad$ $x=3$

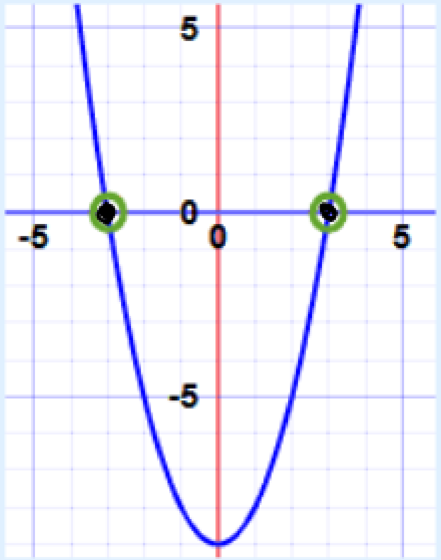
$x^2 - 9 = 0$

$+9 \quad +9$

$\sqrt{x^2} = \sqrt{9}$

$x = \pm 3$

So the roots are -3 and $+3$



std. form ax^2+bx+c

factored form = (x)

And there is something else of interest:

- A polynomial **can be rewritten like this**:

roots (zeros)

any polynomial → $a(x-r_1)(x-r_2)(x-r_3)\ldots$

factors

The factors like $(x-r_1)$ are called **Linear Factors**, because they make a **line** when we plot them.

Example: $x^2 - 9$

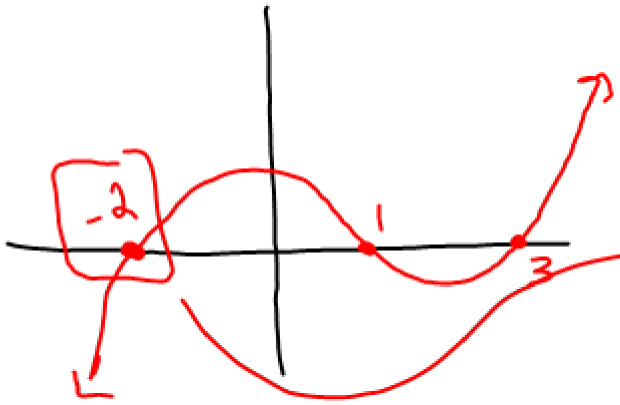
The roots are $r_1 = -3$ and $r_2 = +3$ (as we discovered above) so the factors are:

$$x^2 - 9 = (x+3)(x-3)$$

(in this case **a** is equal to **1** so I didn't put it in)

The Linear Factors are **(x+3)** and **(x−3)**

So knowing the **roots** means we also know the **factors**.



$-2, 1, 3$: "roots"

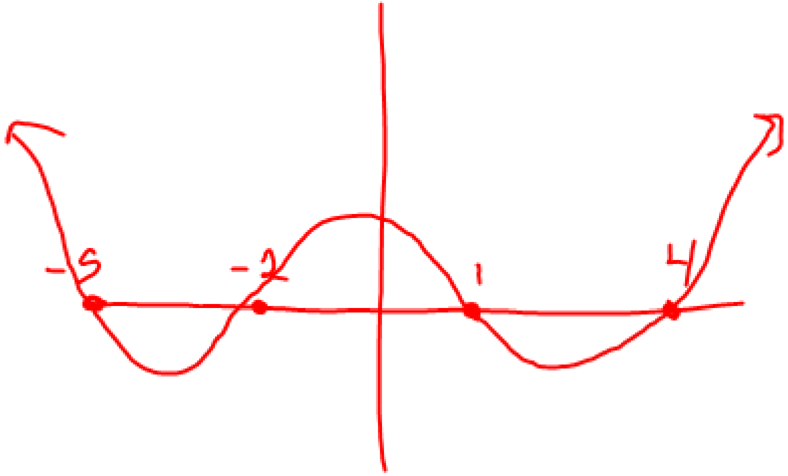
$y = (x+2)(x-1)(x-3)$

$0 = (x+2)$ $\quad$ $0 = (x-1)$

$-2 \qquad -2$

$-2 = x$

$0 = (x-3)$



$y = (x+5)(x+2)(x-1)(x-4)$

Example: $3x^2 - 12$

It is degree 2, so there are 2 roots.

Let us find the roots: We want it to be equal to zero:

$3x^2 - 12 = 0$ (leading coefficient)

$+12 \quad +12$

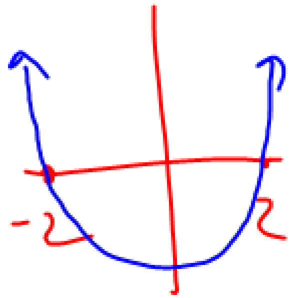
$\frac{3x^2}{3} = \frac{12}{3}$

$\sqrt{x^2} = \sqrt{4}$

$x = \pm 2$

$x = -2 \qquad x = 2$

$y = 3(x+2)(x-2)$



So the roots are:

$$x = -2 \text{ and } x = +2$$

And so the factors are:

$$3x^2 - 12 = 3(x+2)(x-2)$$

Likewise, when we know the **factors** of a polynomial we also know the **roots**.

Example: $3x^2 - 18x + 24$

It is degree 2 so there are 2 factors.

$$3x^2 - 18x + 24 = a(x-r_1)(x-r_2)$$

$$3(x^2 - 6x + 8)$$

$$3(x-4)(x-2)$$

$x = 4$ $x = 2$

(8, −4, −2, −6)

Complex Numbers

We **may** need to use Complex Numbers to make the polynomial equal to zero.

A Complex Number is a combination of a Real Number and an Imaginary Number

And here is an example:

Example: $x^2 - x + 1$

Can we make it equal to zero?

$a = 1 \quad b = -1 \quad c = 1$

$$x^2 - x + 1 = 0$$

1

−1

Not factorable.

$$\frac{1 \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$$

"discriminant"

$$\frac{1 \pm \sqrt{-3}}{2}$$

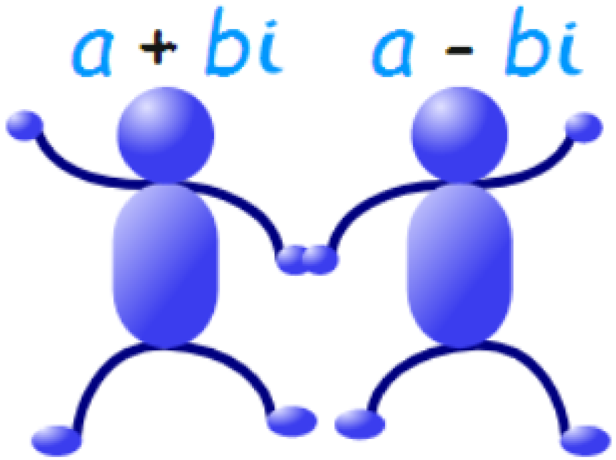
$$\frac{1 \pm i\sqrt{3}}{2}$$

Complex Pairs

So the roots r_1, r_2, ... etc may be Real or Complex Numbers.

But there is something interesting...

Complex Roots **always come in pairs**!



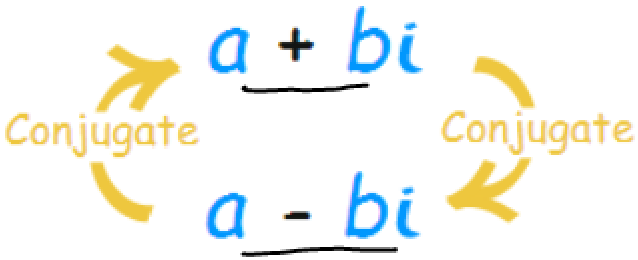
You saw that in our example above:

Example: $x^2 - x + 1$

Has these roots:

$0.5 - 0.866\mathbf{i}$ and $0.5 + 0.866\mathbf{i}$

The pair are actually complex conjugates (where we **change the sign in the middle**) like this:



Always in pairs? Yes

So we either get:

- **no** complex roots
- **2** complex roots
- **4** complex roots,
- etc

And **never** 1, 3, 5, etc.

Which means we automatically know this:

Degree	Roots	Possible Combinations
1	1	1 Real Root
2	2	2 Real Roots, **or** 2 Complex Roots
3	3	3 Real Roots, **or** 1 Real and 2 Complex Roots
4	4	4 Real Roots, **or** 2 Real and 2 Complex Roots, **or** 4 Complex Roots
etc		etc!

5 5 real roots; 3 real 2 imag. ; 1 real 4 imag

And so:

When the degree is odd (1, 3, 5, etc) there is **at least one real root** ... guaranteed!

Example: $3x-6$

The degree is 1.

There is one real root

:

You can actually see that it **must go through the x-axis** at some point.

According to the Fundamental Theorem of Algebra, how many roots does this function have?

$$y = x^3 + 5x^2 - x - 5$$

What type of roots are there?

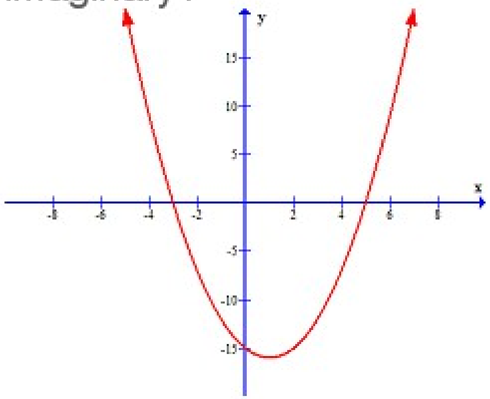
How many of each type of root would there be?

The factored form of a polynomial function is $y = (x + 4)(x - 2)(x - 1)(x + 1)$.

According to the Fundamental Theorem of Algebra, what is the degree of this function?

What are the roots of this function?

The following graph is of a polynomial function of degree 2. Are the solutions of this function real or imaginary?



HW #1: Fundamental Theorem of Algebra