Arithmetic

Warmup:

- Write an explicit and recursive formula for the sequence below.
- Find the 20th term.

+8 +8 +8 +8

4, 12, 20, 28, 36, ...

a_0 a_1

recursive

$$\begin{cases} a_1 = \text{value of 1st term} \\ a_n = a_{n-1} + d \end{cases}$$

$$\begin{cases} a_1 = 12 \\ a_n = a_{n-1} + 8 \end{cases}$$

explicit

$a_n = d(n-1) + a_1$ or $a_n = d \cdot n + a_0$

$a_n = 8(n-1) + 12$ or $a_n = 8n + 4$

term 20

$a_{20} = 8(20-1) + 12$

$8(19) + 12$

$152 + 12$

$a_{20} = 8(20) + 4$

$= 160 + 4$

$= 164$

Unit 4:

Modeling and Analyzing Exponential Functions

EQ: What is a geometric (multiplication)

Problem: Your room is too cold so you decide to adjust the thermostat. The current temperature of the room is 60º Fahrenheit. In an attempt to get warmer, you increase the temperature by 10% every hour. An hour later, it's still not warm enough, so you increase it by 10% again. When this still isn't effective, you continuously increase the temperature in this manner.

Write the sequence of temperatures (round to the nearest 10: ______ 60 + 6 ______
Can you write a formula to generate this sequence?

6 6.6 7.3 8

60, 66, 72.6, 79.9, 87.9

×1.10 ×1.10 ×1.10 ×1.10

$60(1 + .10)$

$60(1.10)$

$60 \times 10\% = 6$ $60 + 6 = 66$

$66 \times 10\% = 6.6$

$72.6 \times .10 = 7.26$ $72.6 + 7.26 = 79.86$

$72.6 \times 1.10 = 79.86$

$79.9 \times 1.10 = 87.9$

add 4

−1, 3, 7, 11, 15, 19

0 term

$y = 4 \cdot x - 1$

4, 8, 16, 32, 64, 128, ...

multiplying by 2.

2^2 2^3 2^4 2^5

$2^1 = 2 \cdot 2$
$2^2 = 4 \cdot 2$
$2^3 = 8 \cdot 2$
$2^4 = 16 \cdot 2$

x	y
1	4
2	8
3	16
4	32
5	64

Geometric Sequence: A sequence of numbers in which the **ratio between any two consecutive terms** is a constant. In other words, it is a sequence of numbers in which you **multiply each term** by a constant to determine the next term.

This constant is called the **common ratio**, represented by the letter **r.**

Explicit Formula:

$$a_n = r^{n-1} \cdot a_1$$

Recursive Formula:

$$\begin{cases} a_1 = \text{value of } 1^{st} \text{ term} \\ a_n = a_{n-1} \cdot r \end{cases}$$

Formulas for our example:

$$a_n = (1.10)^{n-1} \cdot (60)$$

$$\begin{cases} a_1 = 60 \\ a_n = (a_{n-1})(1.10) \end{cases}$$

What is the temperature in the room if we adjust it 12 times? ______________________

term 13

$$a_{13} = (1.10)^{13-1}(60)$$

$$a_{13} = 188.3$$

r = divide any term by its previous term.

Example: Write the recursive and explicit formula for each of the following sequences. Then identify the next 3 terms and the 10th term.

1) 1, 3, 9, 27, 81…

243, 729, 2187

rec

$a_1 = 1$

$a_n = (a_{n-1})(3)$

exp

$a_n = 1 \cdot (3)^{n-1}$

$81 \times 3 = 243$

$243 \times 3 = 729$

$729 \times 3 = 2187$

$a_{10} = 1 \cdot (3)^{10-1} = 19{,}683$

2) 1, -2, 4, -8, 16…

rec

$a_1 = 1$

$a_n = (a_{n-1})(-2)$

exp

$a_n = (1)(-2)^{n-1}$

$16 \times -2 = -32$

$-32 \times -2 = 64$

$64 \times -2 = -128$

$a_{10} = 1(-2)^{10-1} = -512$

3) 4, 2, 1, $\frac{1}{2}$, $\frac{1}{4}$,…

rec

$a_1 = 4$

$a_n = (a_{n-1})(.5)$

exp

$a_n = (4)(.5)^{n-1}$

$\frac{1}{8}, \frac{1}{16}, \frac{1}{32}$

$a_{10} = 4(.5)^{10-1} = \frac{1}{128}$

4) $a_1 = \frac{2}{3}, r = 6$

rec

$a_1 = \frac{2}{3}$

$a_n = a_{n-1} \cdot (6)$

exp

$a_n = (\frac{2}{3})(6)^{n-1}$

4, 24, 144

$a_{10} = (\frac{2}{3})(6)^{10-1}$

$a_{10} = 6{,}718{,}464$

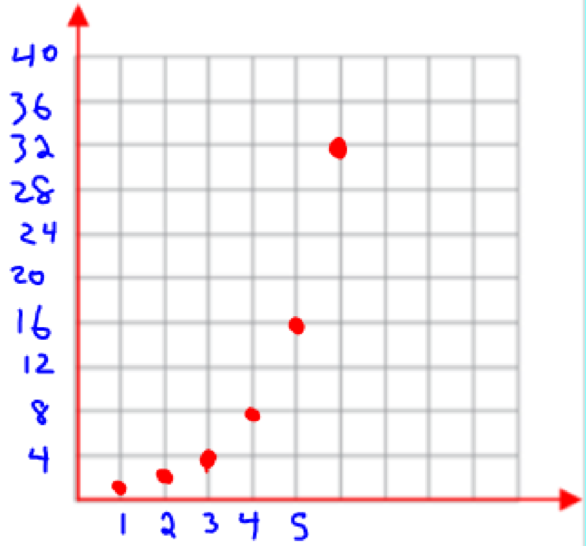
Example: Animals plants, fungi, slime, molds, and other living creatures consist of eukaryotic cells. During growth, generally there is a cell called "mother cell" that divides itself into two "daughter cells." Each of those daughter cells then divides into two more daughter cells, and so on. The sequence to represent the growth is: 1, 2, 4, 8, 16…

Write the explicit formula:

$$a_n = 1(2)^{n-1}$$

Write the recursive formula:

$$\begin{cases} a_1 = 1 \\ a_n = (a_{n-1})(2) \end{cases}$$



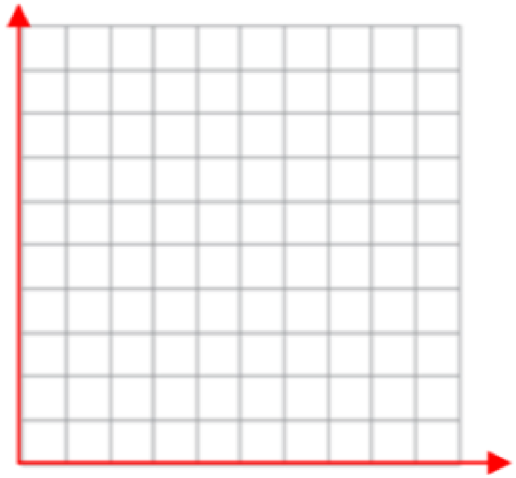
How many cells will there be after the 10^{th} cell division?

$$a_{11} = 1(2)^{11-1}$$

1024

discrete

<u>Example</u>: A mine worker discovers an ore sample containing 500 mg of radioactive material. It is discovered that the radioactive material has a half-life of 1 day. Write the explicit formula for this situation, graph the situation, and find the amount of radioactive material in the sample at the beginning of the 7th day.



HW #1

Geometric Sequences