Warmup:

- Write an explicit and recursive formula for the sequence below.
- Find the 20th term.

$12 - 8 = 4$

$\frac{4}{a_0}$ 12, 20, 28, 36, ...

recursive

$$\begin{cases} a_n = a_{n-1} + \underline{d} \\ a_1 = \text{value of } 1^{st} \text{ term} \end{cases}$$

$$\begin{cases} a_1 = 12 \\ a_n = a_{n-1} + 8 \end{cases}$$

explicit

$$a_n = d \cdot n + a_0$$

or

$$a_n = \underline{d}(n-1) + \underline{a_1}$$

$$a_n = 8(n-1) + 12$$

or

$$a_n = 8n + 4$$

$$a_{20} = 8(20) + 4 \quad \text{or} \quad 8(20-1) + 12$$

$$160 + 4 \qquad\qquad 8(19) + 12$$

$$164 \qquad\qquad 152 + 12$$

$$164$$

Unit 4:

Modeling and Analyzing Exponential Functions

EQ: What is a geometric

Problem: Your room is too cold so you decide to adjust the thermostat. The current temperature of the room is 60° Fahrenheit. In an attempt to get warmer, you increase the temperature by 10% every hour. An hour later, it's still not warm enough, so you increase it by 10% again. When this still isn't effective, you continuously increase the temperature in this manner.

Write the sequence of temperatures (round to the nearest 10: 60, 66, 72.6, 79.9, 87.9

6 6.6 7.3 8

Can you write a formula to generate this sequence?

$60 \times .10 = 6$

$66 \times .10 = 6.6$

$60 + 6 = 66$

$66 + 6.6 = 72.6$

$60 \times 1.10 = 66$

$66 \times 1.10 = 72.6$

$72.6 \times 1.10 = 79.86$

$79.9 \times 1.10 = 87.9$

add

4 8 12 16 ....

$4 \cdot x$

product

$(1.10)^x$

power

Geometric Sequence: A sequence of numbers in which the **ratio between any two consecutive terms** is a constant. In other words, it is a sequence of numbers in which you **multiply each term** by a constant to determine the next term.

This constant is called the **common ratio**, represented by the letter **r.**

Explicit Formula:

$$a_n = (a_1)(r)^{n-1}$$

Recursive Formula:

$$\begin{cases} a_1 = \text{value of } 1^{st} \text{ term} \\ a_n = (a_{n-1})(r) \end{cases}$$

Formulas for our example:

$$a_n = (60)(1.10)^{n-1}$$

$$\begin{cases} a_1 = 60 \\ a_n = (a_{n-1})(1.10) \end{cases}$$

What is the temperature in the room if we adjust it 12 times? ______________________

13th term

$$a_{13} = (60)(1.10)^{13-1} = 188.3$$

common ratio: divide any term by its previous term.

Example: Write the recursive and explicit formula for each of the following sequences. Then identify the next 3 terms and the 10th term.

1) 1, 3, 9, 27, 81…

rec

$a_1 = 1$

$a_n = a_{n-1} \cdot 3$

exp

$a_n = 1(3)^{n-1}$

$81 \times 3 = 243$

$243 \times 3 = 729$

$729 \times 3 = 2187$

$a_{10} = 1(3)^{10-1} = 19683$

2) 1, -2, 4, -8, 16…

rec

$a_1 = 1$

$a_n = a_{n-1} \cdot -2$

exp

$a_n = 1(-2)^{n-1}$

$-32, 64, -128$

$a_{10} = 1(-2)^{10-1} = -512$

3) 4, 2, 1, $\frac{1}{2}$, $\frac{1}{4}$,…

rec

$a_1 = 4$

$a_n = a_{n-1}\left(\frac{1}{2}\right)$

exp

$a_n = 4\left(\frac{1}{2}\right)^{n-1}$

$\frac{1}{8}, \frac{1}{16}, \frac{1}{32}$

$a_{10} = 4\left(\frac{1}{2}\right)^{10-1} = \frac{1}{128}$

4) $a_1 = \frac{2}{3}, r = 6$

rec

$a_1 = \frac{2}{3}$

$a_n = a_{n-1}(6)$

exp

$a_n = \frac{2}{3}(6)^{n-1}$

$4, 24, 144$

$a_{10} = \frac{2}{3}(6)^{10-1}$

$= 6,718,464$

Example: Animals plants, fungi, slime, molds, and other living creatures consist of eukaryotic cells. During growth, generally there is a cell called "mother cell" that divides itself into two "daughter cells." Each of those daughter cells then divides into two more daughter cells, and so on. The sequence to represent the growth is:

y	1, 2, 4, 8, 16...32, 64, 128, ...
x	

Write the explicit formula:

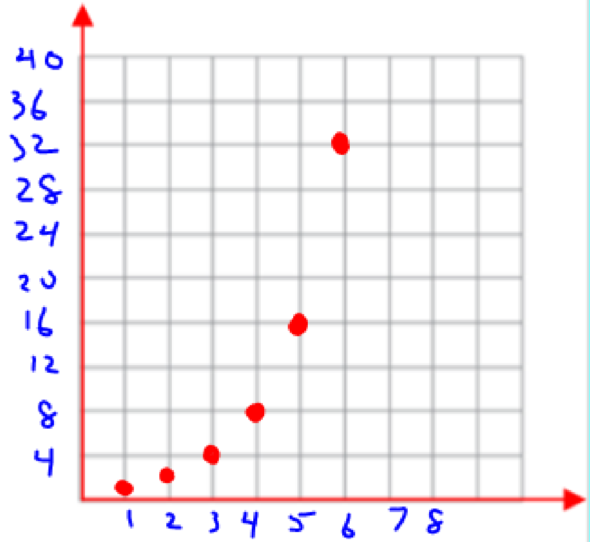
$$\left\{ a_n = 1(2)^{n-1} \right.$$

Write the recursive formula:

$$\left\{ \begin{array}{l} a_1 = 1 \\ a_n = a_{n-1} \cdot 2 \end{array} \right.$$

How many cells will there be after the 10th cell division?

$$a_{11} = 1(2)^{11-1} = 1024$$

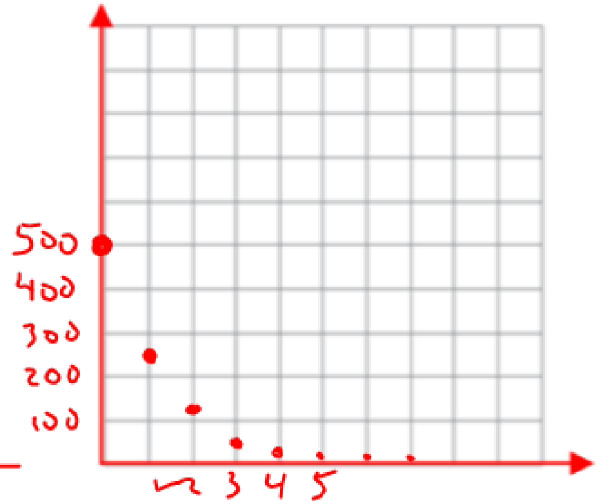


"Discrete"

Example: A mine worker discovers an ore sample containing 500 mg of radioactive material. It is discovered that the radioactive material has a half-life of 1 day. Write the explicit formula for this situation, graph the situation, and find the amount of radioactive material in the sample at the beginning of the 7th day.

500, 250, 125, 62.5, ...

half-life : $r = \frac{1}{2}$



day	0	1	2			
mg	500	250	125			

$$\boxed{a_n = (250)\left(\frac{1}{2}\right)^{n-1}}$$

$$a_7 = 250\left(\frac{1}{2}\right)^{7-1} = \frac{125}{32}$$

$\approx 3.9\,\text{mg}$

HW #1

Geometric Sequences