

Characteristics of Rational Functions

- * **Domain**: all possible "x" values, or input values. [Based on excluded values]
- **Holes**: point of discontinuity. occur when have common factors in numerator & denominator.
- **VA**: pts. of discontinuity. zeros of denominator.
- **HA**: degrees of top/bottom are equal. or, bottom is larger than top. $(y = \text{ratio of leading coefficients})$
- **SA**: degree of top is larger than bottom. Use synthetic division to find S.A.

Intercepts:

x-int: top = 0 & solve
(more than one)

y-int: plug zero in
 for x
[only one]

Positive and Negative Intervals

all x values that make graph be above x-axis. (positive)

all x value where the graph is below x-axis (negative)

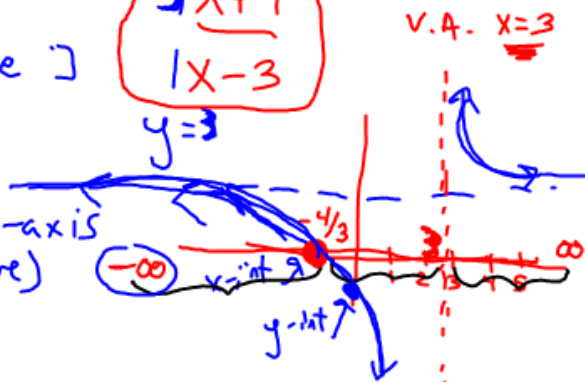
End behavior

As $x \rightarrow -\infty$, $y \rightarrow \frac{3}{1}$
 [Left side]

As $x \rightarrow \infty$, $y \rightarrow \frac{3}{1}$
 [Right side]

S.A.
 $\infty, -\infty$, H.A.

$$\begin{array}{r} 3 \overline{) x+4} \\ x-3 \\ \hline \end{array}$$



$(-\infty, -4/3)$ Positive
 $(-4/3, 3)$ Negative
 $(3, \infty)$ Positive

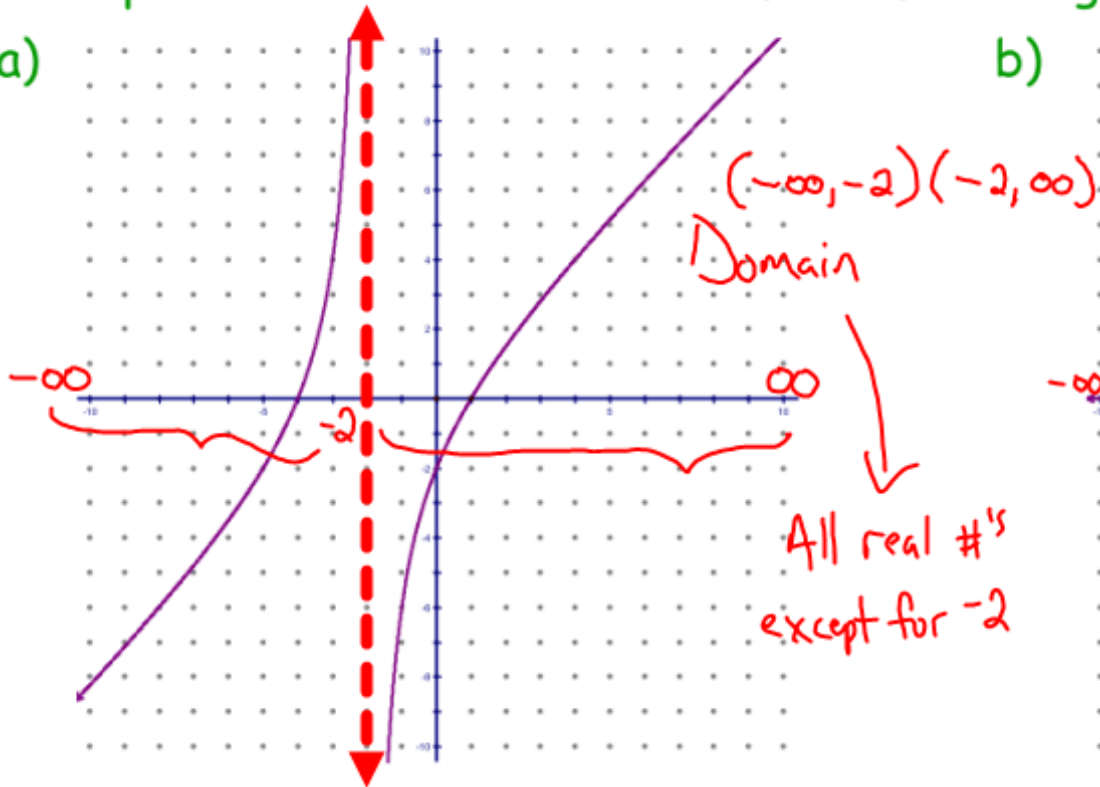
Characteristics of Rational Functions from a Graph

Domain

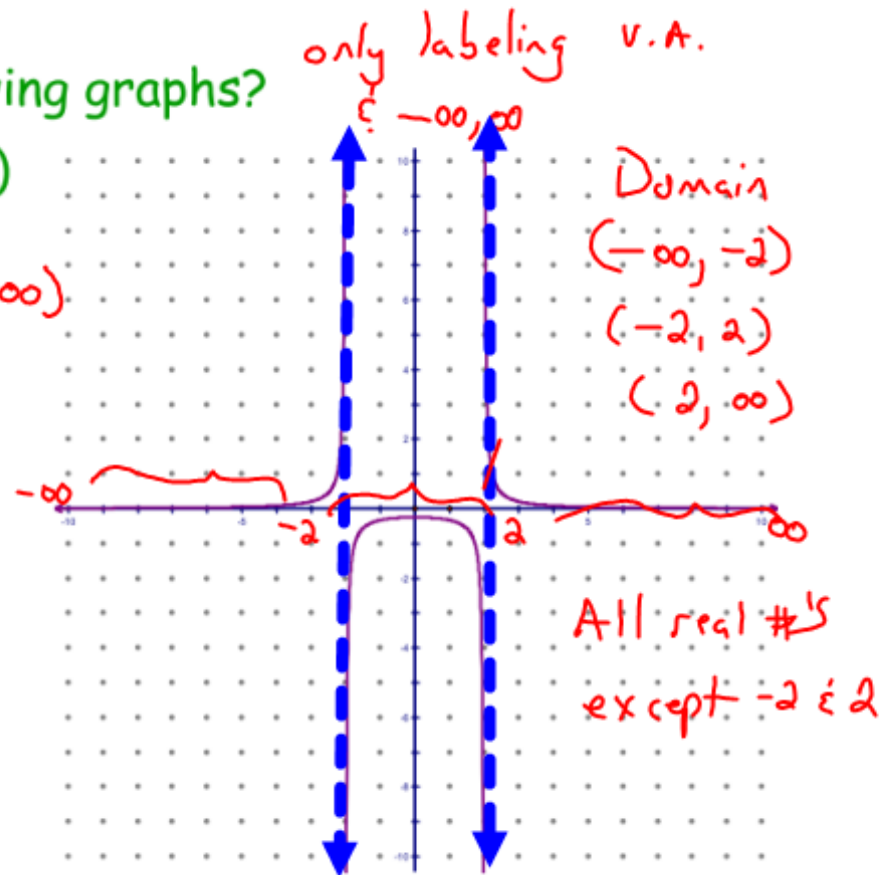
- Look left to right
- List as $x \neq$
 - VA will help!

Example 1: What is the domain of the following graphs?

a)



b)

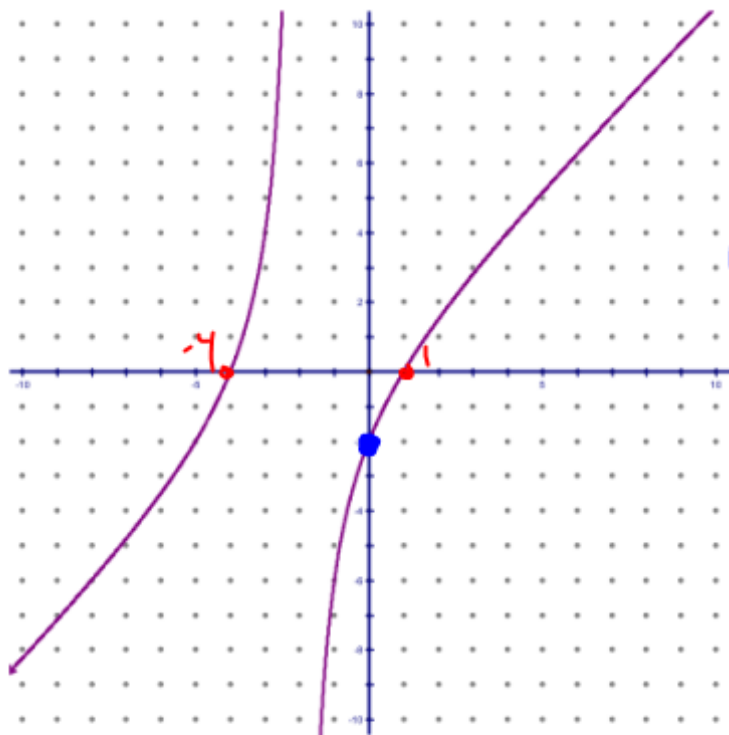


Intercepts

- x-intercepts: look at **x-axis**
- y-intercepts: look at **y-axis**
- list as an **ordered pair**

Example What are the intercepts of the following graphs?

a)



X-ints: $(-4, 0)$ *

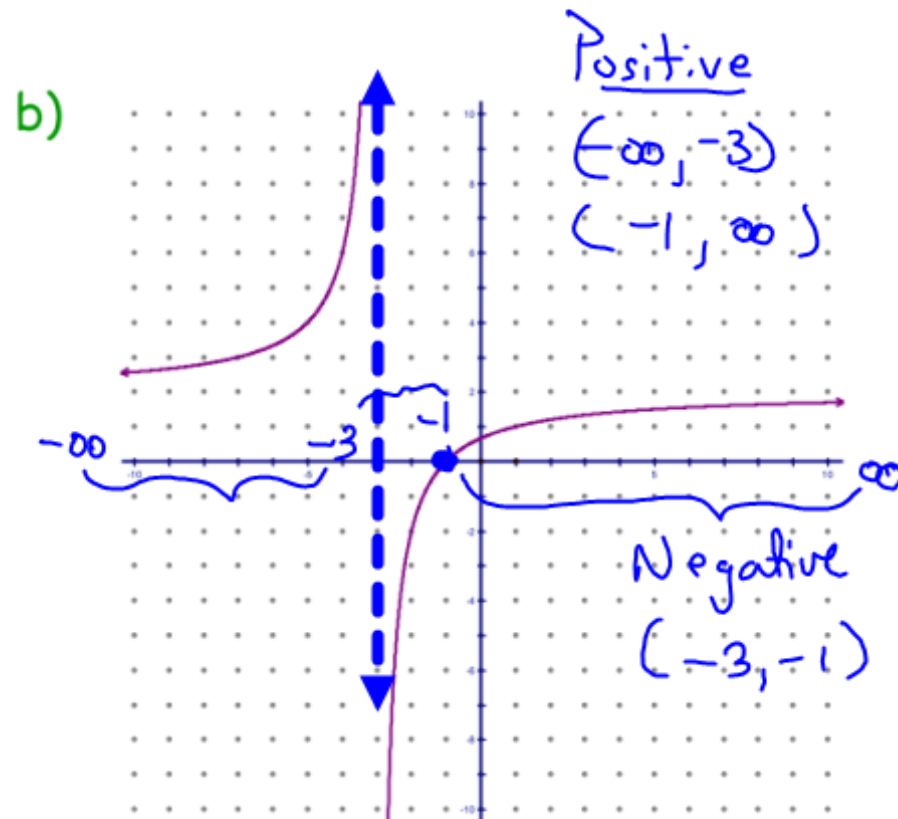
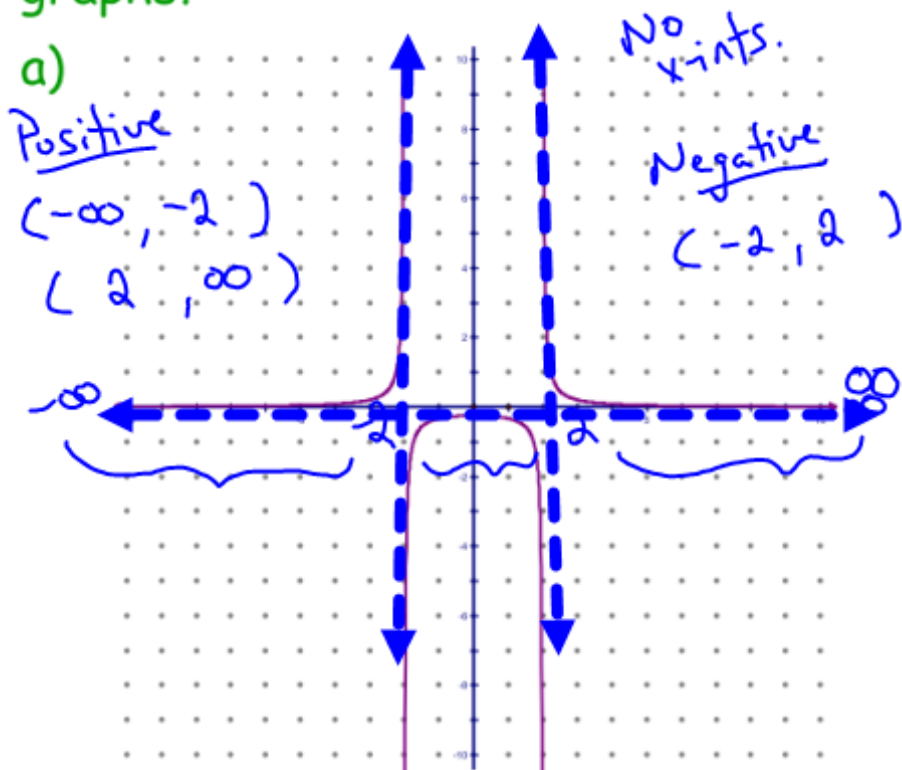
$(1, 0)$ *

y-int: $(0, -2)$ *

Positive and Negative Intervals

- follow the graph from **left to right**
- positive intervals are above the x axis
- negative intervals are below the x axis
- use **x-values** to describe : label $-\infty, \infty$, any x-ints, any v.A.

Example : What are the positive and negative intervals of the following graphs?



End behavior

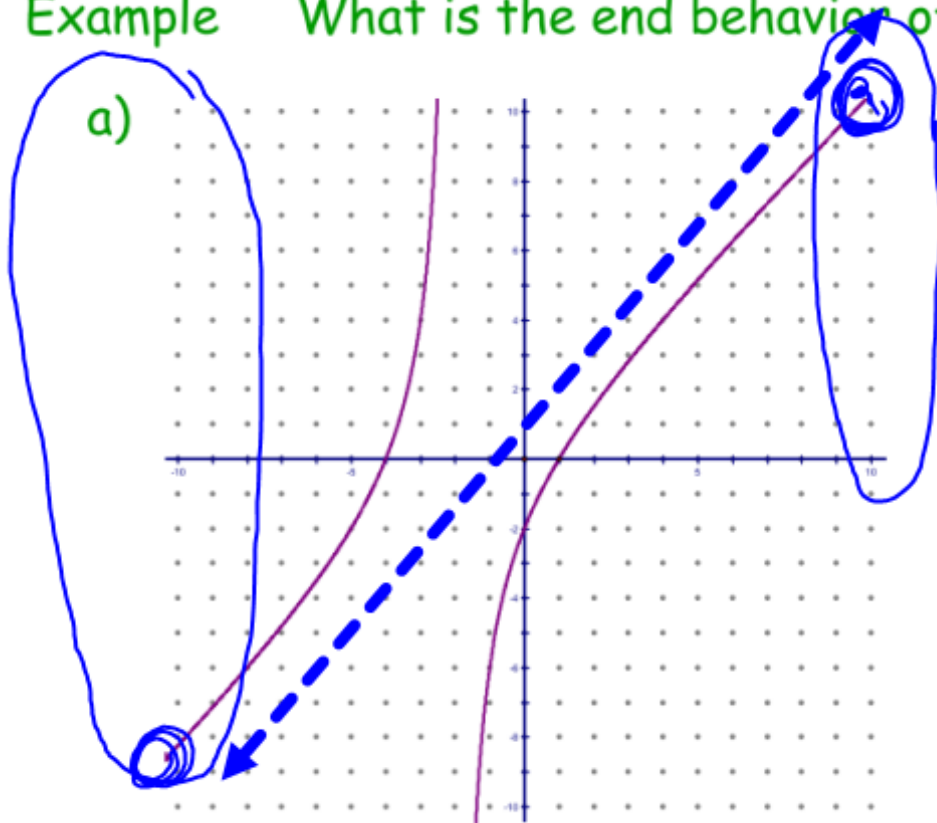
- look at far **left**
 - as x approaches $-\infty$ y approaches $-\infty$
- look at far **right**
 - as x approaches ∞ y approaches ∞

a)

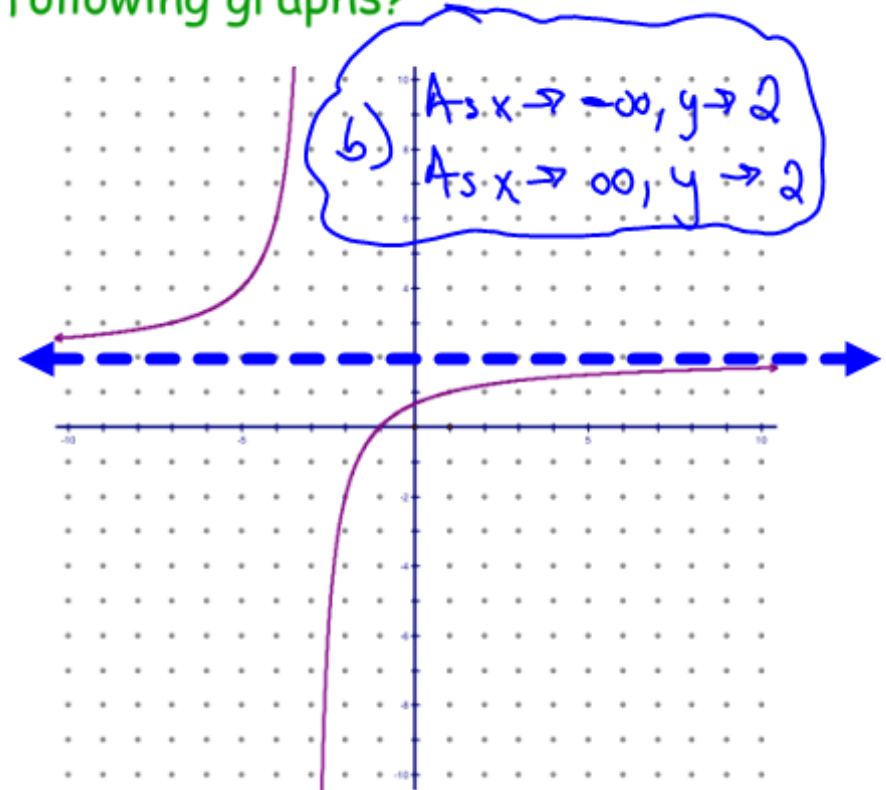
$$\text{As } x \rightarrow -\infty, y \rightarrow -\infty$$

$$\text{As } x \rightarrow \infty, y \rightarrow \infty$$

Example What is the end behavior of the following graphs?



b)



Example Graph $f(x) = \frac{4x-3}{x-1}$ and describe all of its characteristics.

→ Domain: $(-\infty, 1) \cup (1, \infty)$

• Holes: n/a

• VA: $x=1$

• HA: $y=4$

• SA: n/a

• Intercepts:

x-int $(.75, 0)$

y-int $(0, 3)$

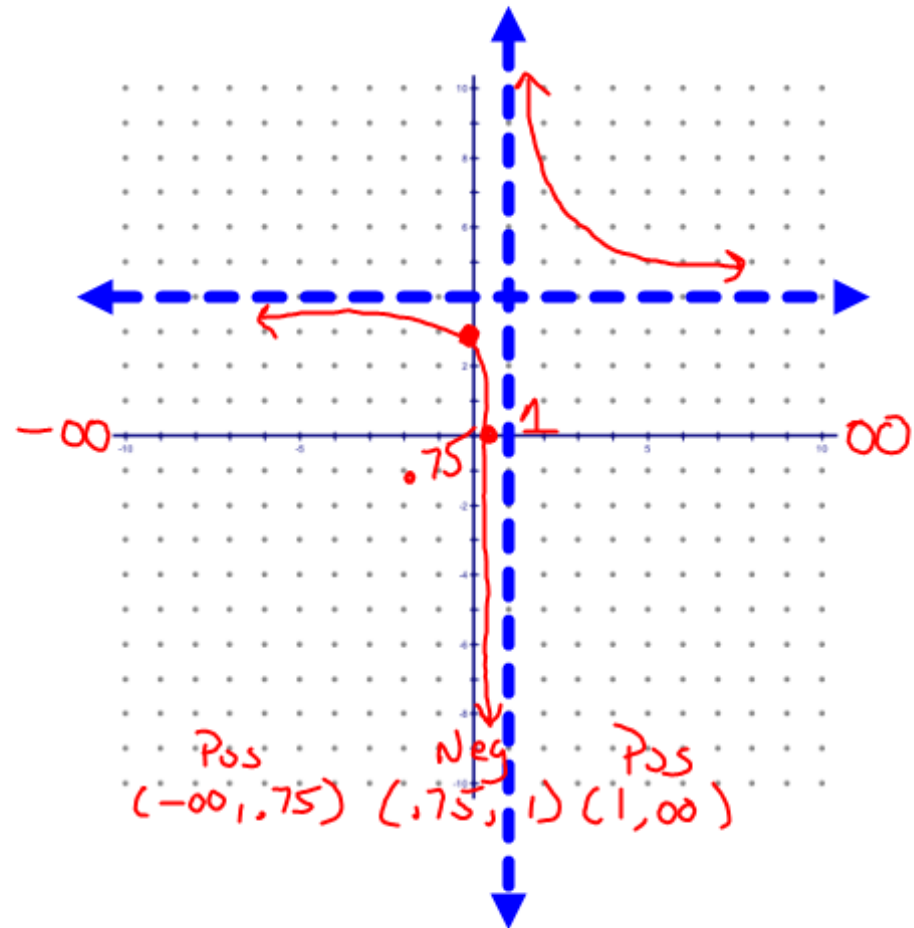
→ Positive and Negative Intervals:

Pos: $(-\infty, .75) \cup (1, \infty)$

Neg: $(.75, 1)$

End behavior: As $x \rightarrow -\infty$, $y \rightarrow 4$

As $x \rightarrow \infty$, $y \rightarrow 4$



$(-\infty, 1) \cup (1, \infty)$ Domain

Homework

Characteristics of
Rational Functions
Worksheet