According to the Fundamental Theorem of Algebra, how many roots does this function have?

3 roots

$$y = x^3 + 5x^2 - x - 5$$

What type of roots are there?

real roots or imaginary numbers. (come in pairs)

How many of each type of root would there be?

3 total –

$\boxed{\text{0 imaginary \& 3 real roots}}$

~~1 imaginary \& 2 real roots~~

$\boxed{\text{2 imaginary \& 1 real roots}}$

~~3 imaginary~~ ~~0 real roots~~

The factored form of a polynomial function is
$y = (x + 4)(x - 2)(x - 1)(x + 1)$.

$x+4=0$

$x-2=0$

$x-1=0$

$x+1=0$

According to the Fundamental Theorem of Algebra, what is the degree of this function? 4

4 factors

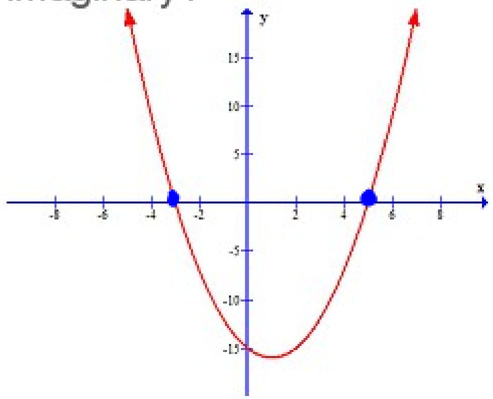
4 roots

Degree = 4

What are the roots of this function?

$x=2$ $x=1$ $x=-4$ $x=-1$

The following graph is of a polynomial function of degree 2. Are the solutions of this function real or imaginary?

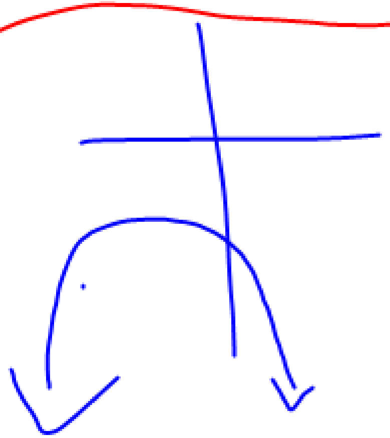


2 roots

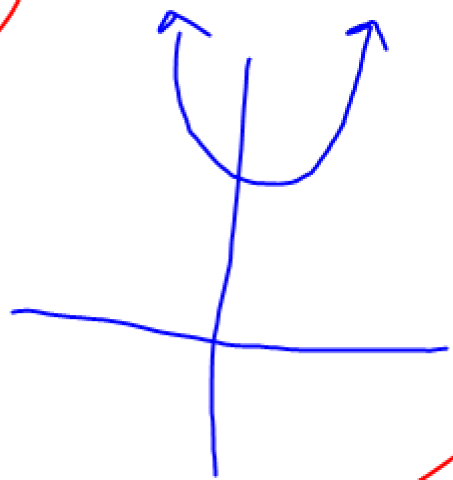
2 real
0 imag.

or

~~2 imag.~~
~~0 real.~~



or



HW #1: Fundamental Theorem of Algebra Answer Key

State the possible number of real and imaginary zeros for each function.

1) $y = x^5 + 5x^4 - x^3 - 5x^2 - 2x - 10$

Possible # of real zeros: 5, 3, or 1
Possible # of imaginary zeros: 4, 2, or 0

2) $y = x^4 - 6x^2 - 27$

Possible # of real zeros: 4, 2, or 0
Possible # of imaginary zeros: 4, 2, or 0

3) $y = x^3 - 5x^2 - x + 5$

Possible # of real zeros: 3 or 1
Possible # of imaginary zeros: 2 or 0

4) $y = x^2 - x - 6$

Possible # of real zeros: 2 or 0
Possible # of imaginary zeros: 2 or 0

Find all zeros.

5) $f(x) = (x + 1)(x^2 - x + 1)$

$\left\{-1, \frac{1 + i\sqrt{3}}{2}, \frac{1 - i\sqrt{3}}{2}\right\}$

6) $f(x) = (x^2 + 5)(x^2 - 7)$

$\{\underline{i\sqrt{5}}, \underline{-i\sqrt{5}}, \sqrt{7}, -\sqrt{7}\}$

$x^2 - 7 = 0$

$\sqrt{x^2} = \sqrt{7}$

$x = \pm\sqrt{7}$

$x^2 + 5 = 0$

$\sqrt{x^2} = \sqrt{-5}$

$x = \pm\sqrt{-5}$

$x = \pm i\sqrt{5}$

7) $f(x) = (x^2 - 5)(x^2 + 7)$

$\{\sqrt{5}, -\sqrt{5}, i\sqrt{7}, -i\sqrt{7}\}$

8) $f(x) = (x^2 + 8)(x^2 + 7)$

$\{2i\sqrt{2}, -2i\sqrt{2}, i\sqrt{7}, -i\sqrt{7}\}$

9) $f(x) = (x^2 + 4)(x^2 + 6)$

$\{2i, -2i, i\sqrt{6}, -i\sqrt{6}\}$

10) $f(x) = (x + 4)(x^2 - 5)$

$\{-4, \sqrt{5}, -\sqrt{5}\}$

Multiplicity

Sometimes a factor appears more than once. That is its **Multiplicity**.

Example: $x^2-6x+9 = \boxed{(x-3)^2}$

9

$-3 \quad -3$

-6

$y = (x-3)(x-3)$

$x = 3 \qquad x = 3$

$\boxed{\text{3 has a multiplicity of 2}}$

1 distinct root, there are actually 2 roots.

Example: x^2-6x+9

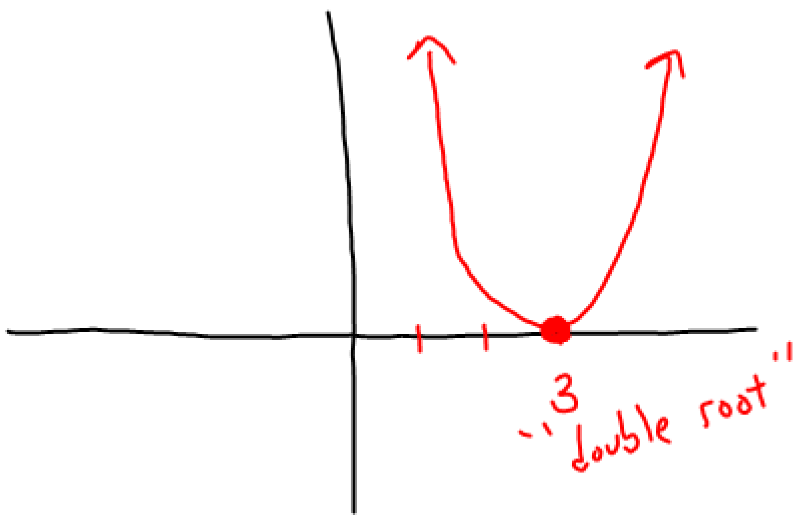
$$x^2-6x+9 = (x-3)(x-3)$$

"(x−3)" appears twice, so the root "3" has **Multiplicity of 2**

degree 2

2 roots

"3"



The **Multiplicities** are included when we say "a polynomial of degree **n** has **n** roots".

Example: x^4+x^3

There **should be** 4 roots (and 4 factors), right?

Factoring is easy, just factor out x^3:

$x^4 + x^3$

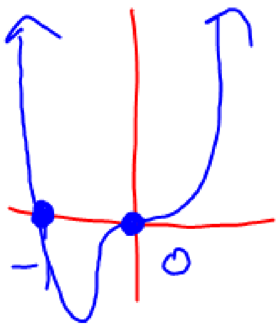
G.C.F.
x^3

$(x^3)\cdot(x+1)$

$(x)\cdot(x)\cdot(x)(x+1)=0$

$x=0 \quad x=0 \quad x=0 \qquad x+1=0$

$x=-1$



multiplicity of 3

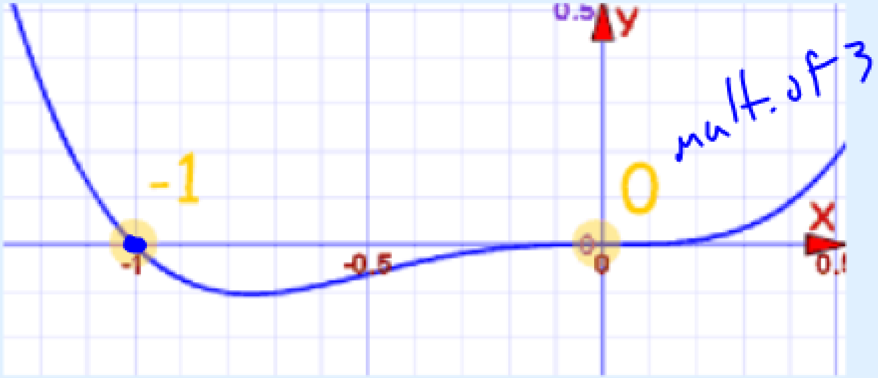
2 distinct roots: -1 & 0

4 roots: $-1, 0, 0, 0$

$$x^4+x^3 = x^3(x+1) = x \cdot x \cdot x \cdot (x+1)$$

there are 4 factors, with "x" appearing 3 times.

But there seem to be only 2 roots, at $\mathbf{x=-1}$ and $\mathbf{x=0}$:



But counting Multiplicities there are actually 4:

- "x" appears three times, so the root "0" has a **Multiplicity of 3**
- "x+1" appears once, so the root "−1" has a **Multiplicity of 1**

Total = 3+1 = 4

Quadratic

Zeros at 3, -4

Write in factored form

$$y = (x-3)(x+4)$$

$$x = -4$$
$$+4 \quad +4$$
$$\boxed{x+4} = 0$$

$$x - 3 = 0$$
$$x = 3$$

Write the equation in factored form given the following information:

zeros at 3 and -5

degree is 3

Root of 3 has a multiplicity of 2

$y = (x-3)(x+5)(x-3)$

$y = (x+5)(x-3)^2$

$x + \cancel{3} = 0$
$\cancel{-3} \quad -3$

$x = 3$
$-3 \quad -3$

$x - 3 = 0$

$x = -5$
$+5 \quad +5$

$x + 5 = 0$

Write the equation in factored form given the following information:

zeros at 0, -2 and 4

degree is 5

Root of 0 has a multiplicity of 3

$y = (x)(x)(x)(x+2)(x-4)$

$y = (x+2)(x-4) \cdot x^3$

$y = x^3(x+2)(x-4)$

$\boxed{x} = 0$ $\quad x = -2 \quad x = 4$

$x + 0 = 0 \quad x+2 \quad x-4$

Write the equation in factored form given the following information:

zeros at -1 and 2

degree is 4

Root of -1 has a multiplicity of 3

$$y = (x+1)^3(x-2)$$

Write the equation in factored form given the following information:

zeros at 7, -3 and 8

degree is 3

$y = (x - 7)(x + 3)(x - 8)$

Fundamental Theorem of Algebra

The "Fundamental Theorem of Algebra" is **not** the start of algebra or anything, but it does say something interesting about polynomials :

Any polynomial of degree **n** has **n** roots
but we may need to use complex numbers

Summary

- A polynomial of degree **n** has **n** roots (where the polynomial is zero)
- A polynomial can be factored like: $\boldsymbol{a(x-r_1)(x-r_2)\ldots}$ where r_1, etc are the roots
- Roots may need to be **Complex Numbers**
- Complex Roots **always come in pairs**
- Sometimes a factor appears more than once. That is its **Multiplicity**.

What are the roots of the equation $5x^2 - 5x - 30 = 0$?

$$5x^2 - 5x - 30 = 0$$

$$5 \cdot (x^2 - x - 6) = 0$$

$$5(x+2)(x-3) = 0$$

$5 \neq 0$ $\quad x = -2$ $\quad x = 3$

-6

2 -3

-1

What are the roots of the equation $x^2 - 6x + 13 = 0$?

$a = 1 \quad b = -6 \quad c = 13$

$$\frac{6 \pm \sqrt{(-6)^2 - 4(1)(13)}}{2(1)}$$

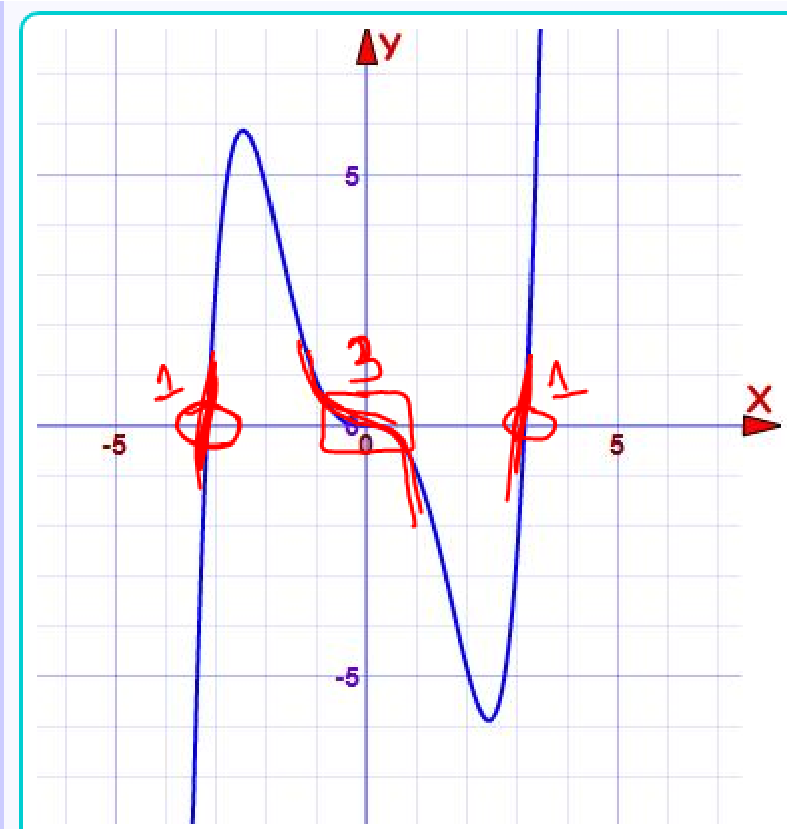
$$\frac{6 \pm \sqrt{36 - 52}}{2}$$

$$\frac{6 \pm \sqrt{-16}}{2}$$

$$\frac{6 \pm 4i}{2}$$

$$\frac{6}{2} \pm \frac{4}{2}i$$

$$\boxed{3 \pm 2i}$$



The above shows part of the graph of the function $y = 0.1x^5 - x^3$.

You can see 3 roots ... what are their multiplicities (from left to right)?

Roots : 5

$y = .1x^5 - x^3$

Roots

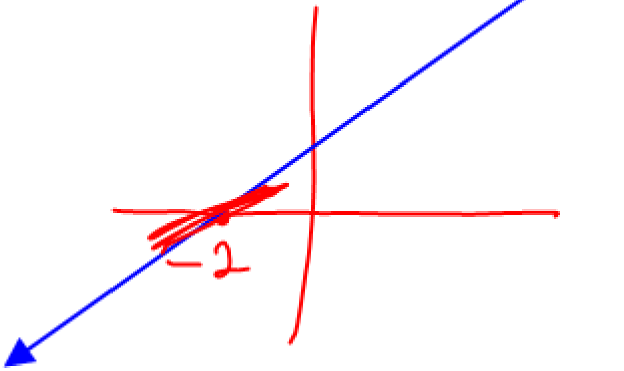
Roots	-3	0	3
mult.	1	3	1

$\boxed{y = 4x + 8}$ root $x = -2$

$4x + 8 = 0$

$\frac{4x}{4} = \frac{-8}{4}$

$x = -2$



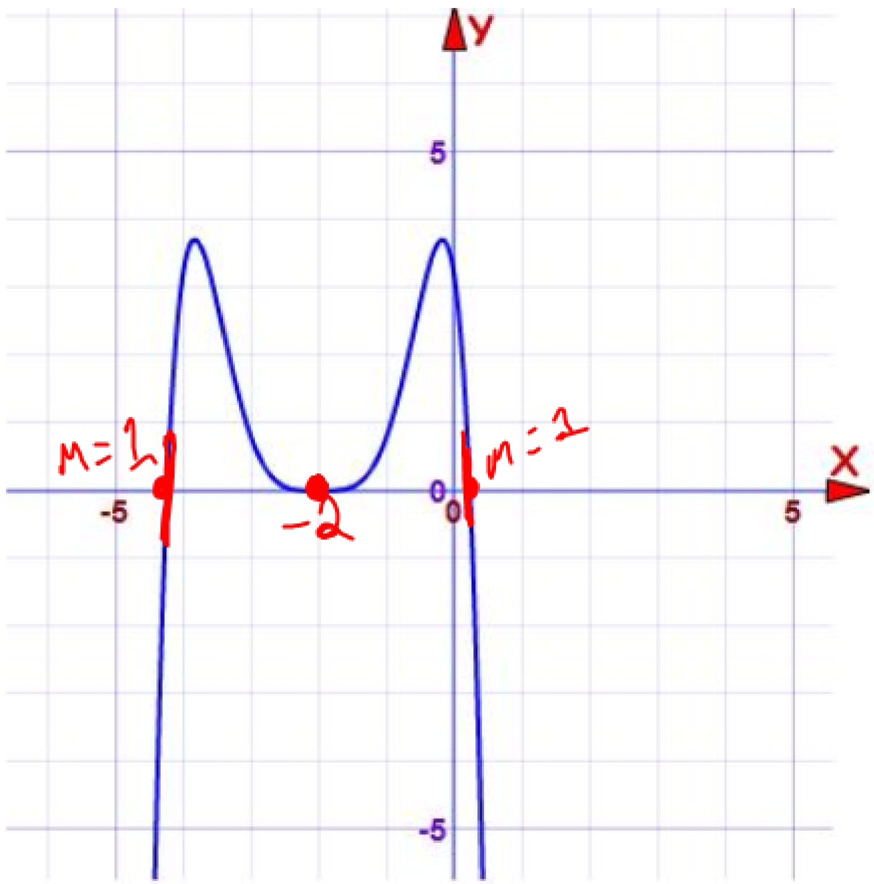
$y = (x+3)(x+5)$

$x = -3$ $x = -5$

mult. of 1

$y = (x-3)(x-3)$

$x = 3$ mult. of 2



mult. = 4

degree = 6

roots = 6

The above shows part of the graph of the function $\boxed{y = -0.2(x + 2)^6 + (x + 2)^4}$.

What is the multiplicity of the root at x = -2?

What are the roots of the equation $3x^2 + 5x + 4 = 0$?

The equation $x^5 - 2x^4 + 2x^3 = 0$ has: G.C.F. ?

A Five distinct real roots

B Three distinct real roots and two complex roots

C One distinct real root and two complex roots

D One distinct real root and four complex roots

$$x^5 - 2x^4 + 2x^3$$

$$x^3(x^2 - 2x + 2) = 0$$

$$x \cdot x \cdot x \quad (x^2 - 2x + 2)$$

$x = 0$ $x = 0$ $x = 0$

HW #2:
Finding Roots and Multiplicity