Warmup:

Given the following sequences:

- write the recursive formula for each
- write the explicit formula for each
- find the 10th term for each
- what type of sequence are they
- what type of function do they each represent

Sequence 1: -12, -8, -4, 0, ...

Sequence 2: -12, -6, -3, -3/2, ...

Warmup:

Given the following sequences:

- write the recursive formula for each
- write the explicit formula for each
- find the 10th term for each
- what type of sequence are they arithmetic
- what type of function do they each represent linear

$a_{10} = 4(10) + -16$

$40 + -16$

$\boxed{a_{10} = 24}$

Sequence 1: $\underset{a_0}{\underline{-16}}$, -12, -8, -4, 0, ...

rec: $\begin{cases} a_1 = -12 \\ a_n = a_{n-1} + 4 \end{cases}$

exp: $a_n = 4(n-1) + -12$

or

$a_n = 4 \cdot n + -16$

Warmup:

Given the following sequences:

- write the recursive formula for each
- write the explicit formula for each
- find the 10th term for each
- what type of sequence are they Geometric
- what type of function do they each represent "Exponential"

$$a_{10} = -12\left(\tfrac{1}{2}\right)^{10-1}$$

$$a_{10} = -12\left(\tfrac{1}{2}\right)^{9}$$

$$\boxed{a_{10} = -\tfrac{3}{128}}$$

Sequence 2: -12, -6, -3, -3/2, ...

rec: $\begin{cases} a_1 = -12 \\ a_n = a_{n-1} \cdot \left(\frac{1}{2}\right) \end{cases}$

exp: $a_n = -12\left(\frac{1}{2}\right)^{n-1}$

$r = \frac{-6}{-12}$ or $\frac{-3}{-6}$ or $\frac{-3/2}{-3}$

$-12 \cdot \text{some \#} = -6$

$$\frac{-12x}{-12} = \frac{-6}{-12}$$

$x = \frac{1}{2}$

1) $\frac{5}{3}$, 5, 15, 45,...

Geometric

$r = 3$

$a_{10} = 5/3 \cdot 3^{(10-1)}$

$a_{10} = 32805$

rec: $a_1 = 5/3$

$a_n = a_{n-1} \cdot 3$

exp: $5/3 \cdot 3^{n-1}$

2) 1, 4, 9, 16,...

other

$1^2 \; 2^2 \; 3^2 \; 4^2$

x^2

x	y
1	1
2	4
3	9
4	16

3) -45, -61, -77, -93,...

Arithmetic

$d = -16$

$a_{10} = -29 - 16(10)$

$a_{10} = -189$

rec: $a_1 = -45$

$a_n = a_{n-1} - 16$

exp: $a_n = -29 - 16n$

4) -3, 1, 5, 9,...

Arithmetic

$d = 4$

$a_{10} = -7 + 4(10)$

$a_{10} = 33$

rec: $a_1 = -3$

$a_n = a_{n-1} + 4$

exp: $a_n = -7 + 4n$

5) -30, 15, -7.5, 3.75,...

Geometric

$r = -\frac{1}{2}$

$a_{10} = -30(-1/2)^{(10-1)}$

$a_{10} \approx .059$

rec: $a_1 = -30$

$a_n = a_{n-1} \cdot -\frac{1}{2}$

exp: $a_n = -30(-\frac{1}{2})^{n-1}$

For each geometric sequence, write the recursive and explicit formulas. Then find the 8th term.

6) $a_1 = 4,\ r = -3$

rec: $a_1 = 4$

$a_n = a_{n-1} \cdot -3$

exp: $a_n = 4(-3)^{n-1}$

$(8-1)$

$a_8 = 4(-3)$

$a_8 = -8748$

7) $a_1 = -\frac{1}{4},\ r = -12$

rec: $a_1 = -1/4$

$a_n = a_{n-1} \cdot -12$

exp: $a_n = -1/4 \cdot (-12)^{n-1}$

$(8-1)$

$a_8 = -1/4 \cdot (-12)$

$a_8 = 8,957,952$

8) $a_1 = -2,\ r = \frac{1}{6}$

$a_1 = -2$

rec: $a_n = a_{n-1} \cdot (1/6)$

exp: $a_n = -2 \cdot (1/6)^{n-1}$

$(8-1)$

$a_8 = -2(1/6)$

$a_8 = -7.14 \times 10^{-6}$

-7.14 E-6

-.00000714

9) $a_1 = 90,\ r = -\frac{1}{3}$

rec: $a_1 = 90$

$a_n = a_{n-1}(-1/3)$

exp: $a_n = 90(-1/3)^{n-1}$

$(8-1)$

$a_8 = 90(-1/3)$

$a_8 \approx -.04$

10) The end of a spring is pulled as far as it will go and then is released. On the first bounce back it extends 54 cm. On its second bounce back it extends 18 cm. On its third bounce back it extends 6 cm. How long does the spring extend after ~~7, 8, 9, and 10 bounce~~ backs?

54, 18, 6, 2, ...

$r = \frac{18}{54} = \frac{1}{3}$

$\boxed{a_n = 54\left(\frac{1}{3}\right)^{n-1}}$

1	2	3	4	5	6	7	8	9	10
54	18	6	2	2/3	2/9	2/27	2/81	2/243	2/729

$\times\frac{1}{3}$ $\times\frac{1}{3}$ $\times\frac{1}{3}$ $\times\frac{1}{3}$ $\times\frac{1}{3}$ $\times\frac{1}{3}$ $\times\frac{1}{3}$

11) Gordon Moore, co-founder of Intel, suggested that the number of transistors on a square inch of integrated circuit in a computer chip would double every 18 months. If the number of transistors on a square inch of integrated circuit started at 10, find the number of transistors over the course of the next 10 years.

10	20	40	80	160	320	640	1280
	18 months	36 months					
	1.5 yrs	3 yrs	4.5 yrs	6 yrs	7.5 yrs	9 yrs	10 yrs

12) A culture of bacteria doubles every 2 hours. If there are 500 bacteria at the beginning, how many bacteria will there be after 24 hours? Graph this situation.

$n=1$

500, 1000, 2000, 4000, 8000, 16000, 32000, 64000

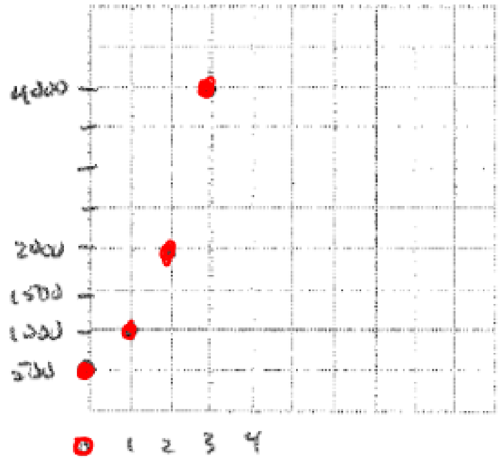
initial, 2hrs, 4hrs, 6hrs, 8hrs, 10hrs, 12hrs, 14hrs

$$\boxed{a_n = \underbrace{1000}(2)^{n-1}}$$

$(n = 12)$

128000, 256000, 512000, 1024000, 2,048,000

16hrs, 18hrs, 20hrs, 22hrs, 24hrs



$$\boxed{500(2)^{n-1}}$$

$(n = 13)$

Describe what's happening in each table below:

x	y
0	2
1	4
2	6
3	8
4	10

adding 2

x	y
0	0
1	1
2	4
3	9
4	16

squaring "x"

x	y
0	2
1	4
2	8
3	16
4	32

mult. by 2

What type of function do each of these tables represent?

x	y
0	2
1	4
2	6
3	8
4	10

Linear

Unit 2

x	y
0	0
1	1
2	4
3	9
4	16

Quadratic

Unit 3

x	y
0	2
1	4
2	8
3	16
4	32

Exponential

Unit 4

Can you complete the table for the listed negative values?

x	y
-3	−4
-2	−2
-1	0
0	2
1	4
2	6
3	8
4	10

−2

+2

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9
4	16

Sq. the x value

1/256
1/128
1/64
1/32
1/16
1/8

x	y
-3	1/4
-2	1/2
-1	1
0	2
1	4
2	8
3	16
4	32

÷2

×2

Can you write the equation that would generate each of these tables?

x	y
-3	-4
-2	-2
-1	0
0	$\boxed{2}$
1	4
2	6
3	8
4	10

+2
+2

$y = mx + b$

$y = 2x + b$

$y = 2x + 2$

x	y
-3	9
-2	4
-1	1
0 vertex	0
1	1
2	4
3	9
4	16

$y = a(x-h)^2 + k$

$y = a(x-0)^2 + 0$

$y = a x^2$ $\quad \boxed{y = 1 \cdot x^2}$

$1 = a(1)^2$

$1 = 1a$

$1 = a$

$y = x^2$

x	y
-3	.25
-2	.5
-1	1
0	2
$\boxed{1}$	4
2	8
3	16
4	32

x2
x2
x2

$y = \cancel{4}(2)^{x-1}$

$\boxed{y = 2(2)^x}$

$y = \underset{\text{y-int}}{a} \cdot \underset{\text{(ratio)}}{b}^{x}$

Unit 4:

Modeling and Analyzing Exponential Functions

Exponential Functions

$$y = a \cdot b^x$$

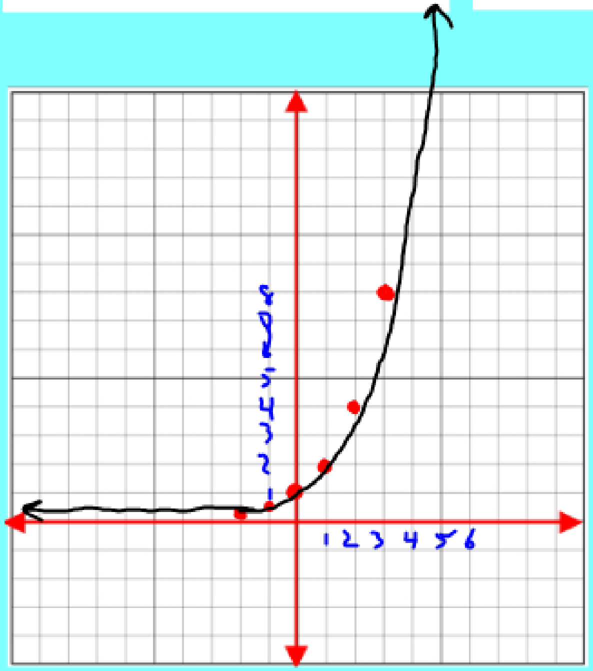
$$f(x) = a \cdot b^x$$

a represents the y-intercept
b represents the "base"

Make a table of values and graph the following 2 functions:

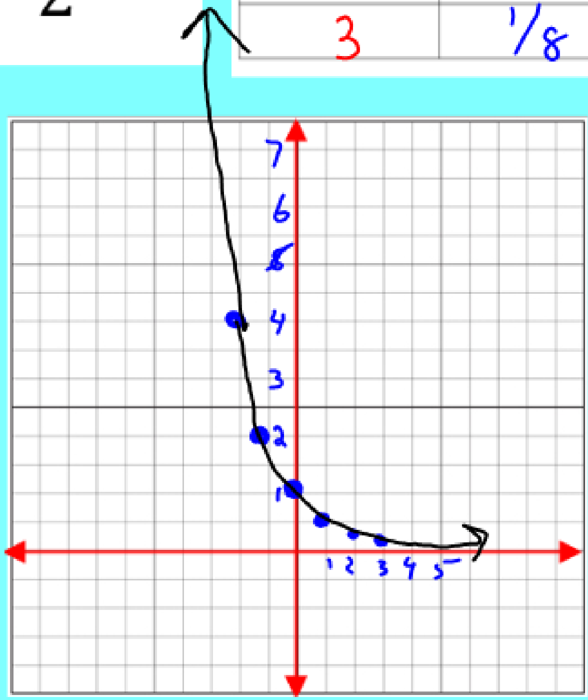
$$y = 2^x$$

x	y
-2	1/4
-1	1/2
0	1
1	2
2	4
3	8



$$y = \left(\frac{1}{2}\right)^x$$

x	y
-2	4
-1	2
0	1
1	1/2
2	1/4
3	1/8



In the previous two examples we see two basic types of exponential functions:

Exponential Growth:

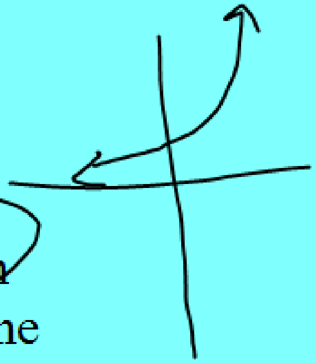
$a > 0 \; and \; b > 1$

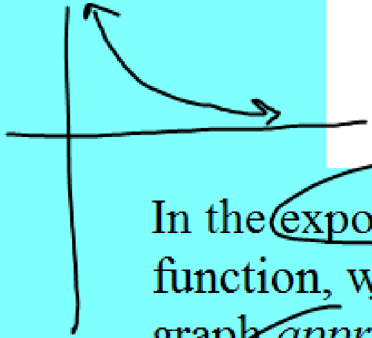
Exponential Decay:

$a > 0 \; and \; 0 < b < 1$

We should also notice that the graphs of each of those two
functions get closer and closer to the x-axis, or a

$$y = 2^x$$





$$y = \left(\frac{1}{2}\right)^x$$

In the exponential growth function, we notice that the graph *approaches* the x-axis, or a y value of 0 on the left side of the graph.

In the exponential decay function, we notice that the graph *approaches* the x-axis, or a y value of 0 on the right side of the graph.

This line, the x-axis, is called an *asymptote*. Every exponential function will have one. For these functions, we say the asymptote is the line y = 0.

as·ymp·tote

/ˈasəm(p)ˌtōt/

noun

a line that continually approaches a given curve but does not meet it at any finite distance.

We should also notice that each of these two functions have a y-intercept of 1.

That is because the *a* value in each equation is 1.

$$y = 2^x$$

$y = 1 \cdot (2)^x$

$(0, 1)$

$y = 1 \cdot (\frac{1}{2})^x$

$$y = (\frac{1}{2})^x$$

$(0, 1)$

The final thing we should notice is that each of the functions has the following point on its graph:

(1,b)

$(1, a \cdot b)$

$y = 2^x$

$y = \left(\frac{1}{2}\right)^x$

$(1,2)$

$\left(1, \frac{1}{2}\right)$

If the *a* value is 1, the point (1,b) will be on the graph.

Make a table of values and graph the following 2 functions:

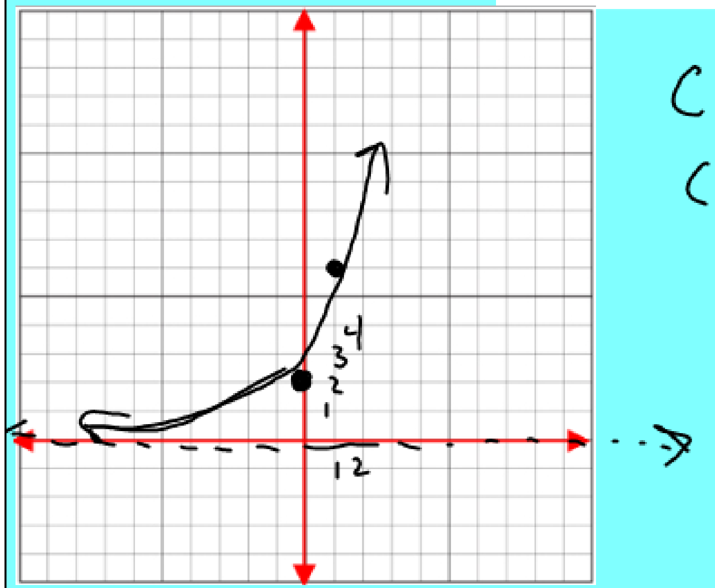
(1, ab)

(1, 2·1/3)

y-int

$$y = \boxed{2} \cdot 3^x$$

x	y
-2	
-1	2/3
0	2
1	6
2	18
3	54



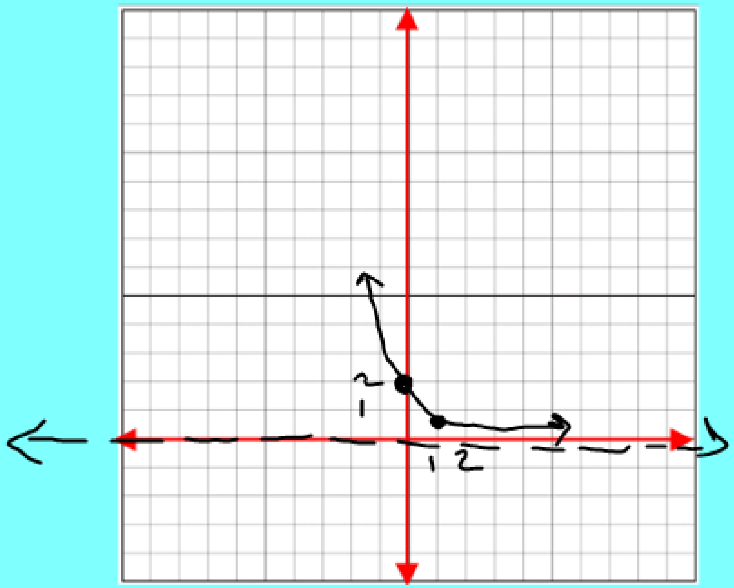
(1, a·b)

(1, 2·3)

(1, 6)

$$y = 2 \cdot \left(\frac{1}{3}\right)^x$$

x	y
0	2
1	2/3



We should see most of the same things on the graphs of these two functions:

$$y = 2 \cdot 3^x$$

- exponential growth
- y-intercept of (0,2)
- asymptote of y = 0

$$y = 2 \cdot (\frac{1}{3})^x$$

- exponential decay
- y-intercept of (0,2)
- asymptote of y = 0

What about the point (1,b)?
Is that point still on our graphs?

What we should notice now is that when a is not equal to 1, the point (1,ab) will be on our graph.

$$y = 2 \cdot 3^x$$

$$(1, 2 \cdot 3)$$

$$(1, 6)$$

$$y = 2 \cdot (\frac{1}{3})^x$$

$$\left(1, 2 \cdot \frac{1}{3}\right)$$

$$(1, \frac{2}{3})$$

To graph exponential functions remember the following:

The y-intercept will be (0,a)

The point (1,ab) is on your graph

- If a = 1, the point will be (1,b)

The asymptote will be the line y = 0

If b > 1, it will represent exponential growth and increase

If 0 < b < 1, it will represent exponential decay and decrease

So, to graph an exponential function, simply find your 2 points, draw your asymptote, and draw your growth or decay function.

Of course this will all be a little different when we introduce transformations tomorrow!

HW #2: Graphing exponential functions