

## Warmup:

Given the following sequences:

- write the recursive formula for each
- write the explicit formula for each
- find the 10th term for each
- what type of sequence are they
- what type of function do they each represent

Sequence 1:  $-12, -8, -4, 0, \dots$

Sequence 2:  $-12, -6, -3, -3/2, \dots$



## Warmup:

Given the following sequences:

- write the recursive formula for each
- write the explicit formula for each
- find the 10th term for each
- what type of sequence are they *Geometric*
- what type of function do they each represent *Exponential.*

$$a_{10} = -12 \left(\frac{1}{2}\right)^{10-1}$$

$$a_{10} = -\frac{3}{128}$$

Sequence 2:  $-12, -6, -3, -3/2, \dots$

$$\text{rec: } \begin{cases} a_1 = -12 \\ a_n = a_{n-1} \cdot \left(\frac{1}{2}\right) \end{cases}$$

$$\begin{aligned} \text{exp: } a_n &= (-12) \left(\frac{1}{2}\right)^{n-1} \\ a_n &= -12 \left(\frac{1}{2}\right)^{n-1} \\ &= -12 \cdot \frac{1}{2}^{n-1} \end{aligned}$$

1)  $\frac{5}{3}, 5, 15, 45, \dots$

Geometric

$r = 3$

$a_{10} = \frac{5}{3} \cdot 3^{(10-1)}$

$a_{10} = 32805$

rec:  $a_1 = \frac{5}{3}$

$a_n = a_{n-1} \cdot 3$

exp:  $\frac{5}{3} \cdot 3^{n-1}$

2) 1, 4, 9, 16, ...

1 2 3 4  
otherQuadratic

$n^2$

3) -45, -61, -77, -93, ...

Arithmetic

$d = -16$

$a_{10} = 29 - 16(10)$

$a_{10} = -189$

rec:  $a_1 = -45$

$a_n = a_{n-1} - 16$

exp:  $a_n = -29 - 16n$

4) -3, 1, 5, 9, ...

Arithmetic

$d = 4$

$a_{10} = -7 + 4(10)$

$a_{10} = 33$

rec:  $a_1 = -3$

$a_n = a_{n-1} + 4$

exp:  $a_n = -7 + 4n$

5) -30, 15, -7.5, 3.75, ...

Geometric

$r = -\frac{1}{2}$

$a_{10} = -30 \left(-\frac{1}{2}\right)^{(10-1)}$

$a_{10} \approx .059$

rec:  $a_1 = -30$

$a_n = a_{n-1} \cdot -\frac{1}{2}$

exp:  $a_n = -30 \left(-\frac{1}{2}\right)^{n-1}$

For each geometric sequence, write the recursive and explicit formulas. Then find the 8<sup>th</sup> term.

6)  $a_1 = 4, r = -3$

rec:  $a_1 = 4$   
 $a_n = a_{n-1} \cdot -3$

exp:  $a_n = 4(-3)^{n-1}$

$a_8 = 4(-3)^{(8-1)}$

$a_8 = -8748$

8)  $a_1 = -2, r = \frac{1}{6}$

$a_1 = -2$

rec:  $a_n = a_{n-1} \cdot (\frac{1}{6})$

exp:  $a_n = -2 \cdot (\frac{1}{6})^{n-1}$

$a_8 = -2(\frac{1}{6})^{(8-1)}$

$a_8 = -7.14 \times 10^{-6}$

scientific notation

$-7.14 \dots \times 10^{-6}$

$-0.00000714$

7)

$a_1 = -\frac{1}{4}, r = -12$

rec:  $a_1 = -\frac{1}{4}$

$a_n = a_{n-1} \cdot -12$

exp:  $a_n = -\frac{1}{4} \cdot (-12)^{n-1}$

$a_8 = -\frac{1}{4} \cdot (-12)^{(8-1)}$

$a_8 = -\frac{1}{4} \cdot (-12)$

$a_8 = 8,957,952$

9)

$a_1 = 90, r = -\frac{1}{3}$

rec:  $a_1 = 90$

$a_n = a_{n-1} \cdot (-\frac{1}{3})$

exp:  $a_n = 90(-\frac{1}{3})^{n-1}$

$a_8 = 90(-\frac{1}{3})^{(8-1)}$

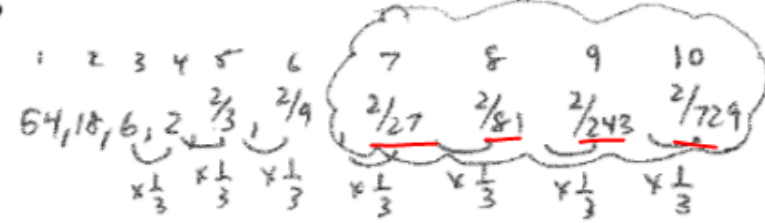
$a_8 = 90(-\frac{1}{3})$

$a_8 = -0.04$

10) The end of a spring is pulled as far as it will go and then is released. On the first bounce back it extends 54 cm. On its second bounce back it extends 18 cm. On its third bounce back it extends 6 cm. How long does the spring extend after 7, 8, 9, and 10 bounce backs?

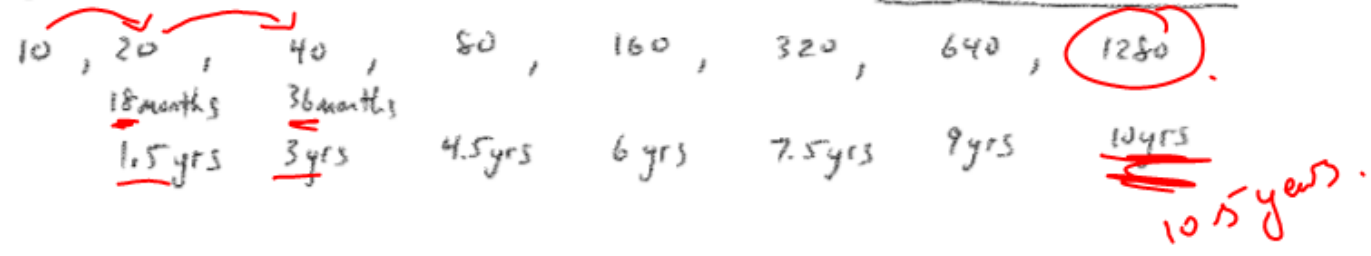
54, 18, 6, 2, ...

$$a_n = 54\left(\frac{1}{3}\right)^{n-1}$$



$$r = \frac{18}{54} = \frac{1}{3}$$

11) Gordon Moore, co-founder of Intel, suggested that the number of transistors on a square inch of integrated circuit in a computer chip would double every 18 months. If the number of transistors on a square inch of integrated circuit started at 10, find the number of transistors over the course of the next 10 years.





Describe what's happening in each table below:

x	y
0	2
1	4
2	6
3	8
4	10

x	y
0	0
1	1
2	4
3	9
4	16

x	y
0	2
1	4
2	8
3	16
4	32



What type of function do each of these tables represent?

x	y
0	2
1	4
2	6
3	8
4	10

x	y
0	0
1	1
2	4
3	9
4	16

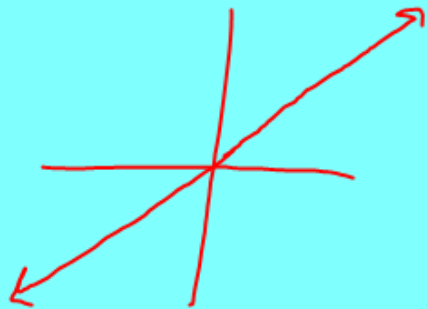
x	y
0	2
1	4
2	8
3	16
4	32

Can you complete the table for the listed negative values?

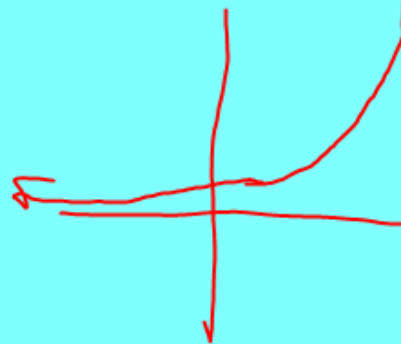
x	y
-3	-4
-2	-2
-1	0
0	2
1	4
2	6
3	8
4	10

-2

+2



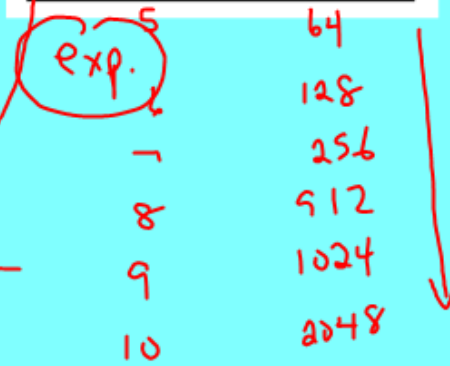
x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9
4	16



x	y
-3	1/4
-2	1/2
-1	1
0	2
1	4
2	8
3	16
4	32

÷2

x2



-9    1/256  
 -8    1/128  
 -7    1/64  
 -6    1/32  
 -5    1/16  
 -4    1/8

exp.  
 5    64  
 6    128  
 7    256  
 8    512  
 9    1024  
 10    2048

Can you write the equation that would generate each of these tables?

x	y
-3	-4
-2	-2
-1	0
0	2
1	4
2	6
3	8
4	10

Linear

$$y = 2x + 2$$

$y = a(x-h)^2 + k$

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9
4	16

Quadratic

$$y = (x)^2$$

$$y = x^2$$

$$y = a(x-0)^2 + 0$$

$$y = ax^2 \quad y = 1x^2$$

$$4 = a(2)^2$$

$$\frac{4}{4} = \frac{4a}{4}$$

$$a = 1$$

x	y
-3	.25
-2	.5
-1	1
0	2
1	4
2	8
3	16
4	32

Exponential

$$a_n = 4(2)^{n-1}$$

$$y = 4(2)^{x-1}$$

$$y = 2(2)^x$$

$$y = a \cdot b^x$$

$$y = mx + b$$

$$y = ax + b$$

$$y = ax^2 + bx + c$$

**Unit 4:**  
**Modeling and  
Analyzing Exponential  
Functions**

# Exponential Functions

$$y = a \cdot b^x$$

$$f(x) = a \cdot b^x$$

$a$  represents the y-intercept

$b$  represents the "base" (ratio)  
rate of multiplication

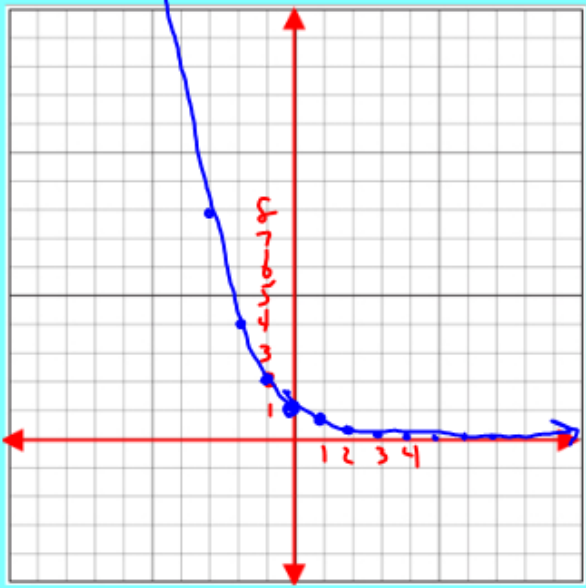
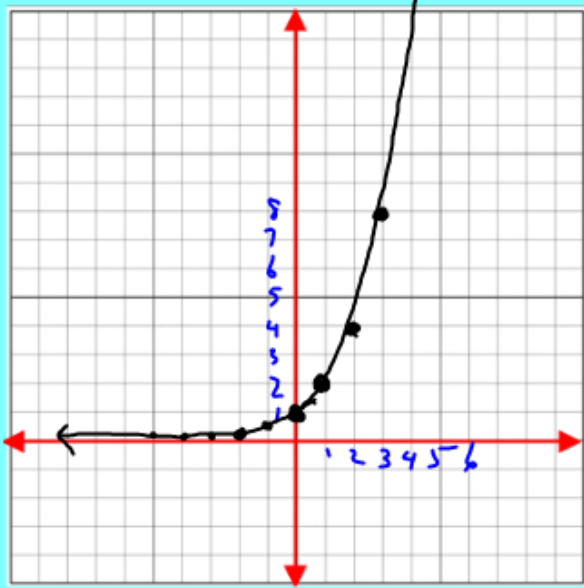
Make a table of values and graph the following 2 functions:

$$y = 2^x$$

x	y
-3	$\frac{1}{8}$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8
4	16
5	32

$$y = \left(\frac{1}{2}\right)^x$$

x	y
-4	16
-3	8
-2	4
-1	2
0	1
1	$\frac{1}{2}$
2	$\frac{1}{4}$
3	$\frac{1}{8}$
4	$\frac{1}{16}$



In the previous two examples we see two basic types of exponential functions:

**Exponential Growth:**

$$a > 0 \text{ and } b > 1$$

**Exponential Decay:**

$$a > 0 \text{ and } 0 < b < 1$$

We should also notice that the graphs of each of those two functions get closer and closer to the x-axis, or a

$$y = 2^x$$

In the exponential growth function, we notice that the graph approaches the x-axis, or a y value of 0 on the left side of the graph.


$$y = \left(\frac{1}{2}\right)^x$$

In the exponential decay function, we notice that the graph approaches the x-axis, or a y value of 0 on the right side of the graph.

This line, the x-axis, is called an asymptote. Every exponential function will have one. For these functions, we say the asymptote is the line  $y = 0$ .



# as·ymp·tote

*/ˈasəm(p), tōt/* 

*noun*

a line that continually approaches a given curve but does not meet it at any finite distance.

We should also notice that each of these two functions have a y-intercept of 1.

That is because the  $a$  value in each equation is 1.

$$y = 1 \cdot 2^x$$

$$y = a \cdot b^x$$
$$y = 1 \cdot 2^x$$

$$y = 1 \cdot \left(\frac{1}{2}\right)^x$$

The final thing we should notice is that each of the functions has the following point on its graph:

(1,b)

1 · 2<sup>1</sup>

$$y = 1 \cdot 2^x$$

$$y = 1 \cdot 2$$

$$y = \left(\frac{1}{2}\right)^x$$

(1,2)

$\left(1, \frac{1}{2}\right)$

If the  $a$  value is 1, the point (1,b) will be on the graph.

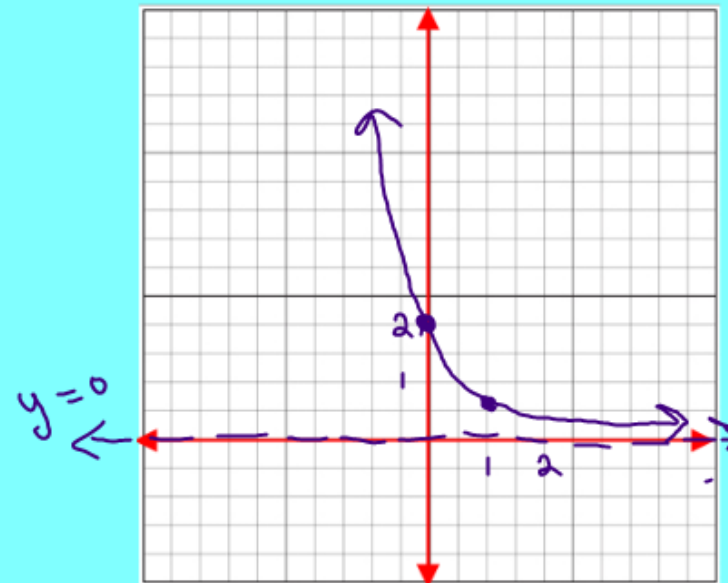
Make a table of values and graph the following 2 functions:

$$y = 2 \cdot 3^x$$

x	y
-2	$\frac{2}{9}$
-1	$\frac{2}{3}$
0	2
1	6
2	18
3	54

$$y = 2 \cdot \left(\frac{1}{3}\right)^x$$

x	y
0	2
1	$\frac{2}{3}$



We should see most of the same things on the graphs of these two functions:

$$y = 2 \cdot 3^x$$

- exponential growth
- y-intercept of (0,2)
- asymptote of  $y = 0$

$$y = 2 \cdot \left(\frac{1}{3}\right)^x$$

- exponential decay
- y-intercept of (0,2)
- asymptote of  $y = 0$

What about the point (1,b)?  
Is that point still on our graphs?

What we should notice now is that when  $a$  is not equal to 1, the point  $(1,ab)$  will be on our graph.

$$y = 2 \cdot 3^x$$

$$(1, 2 \cdot 3)$$

$$(1, 6)$$

$$y = 2 \cdot \left(\frac{1}{3}\right)^x$$

$$\left(1, 2 \cdot \frac{1}{3}\right)$$

$$\left(1, \frac{2}{3}\right)$$

To graph exponential functions remember the following:

The y-intercept will be  $(0, a)$

The point  $(1, ab)$  is on your graph

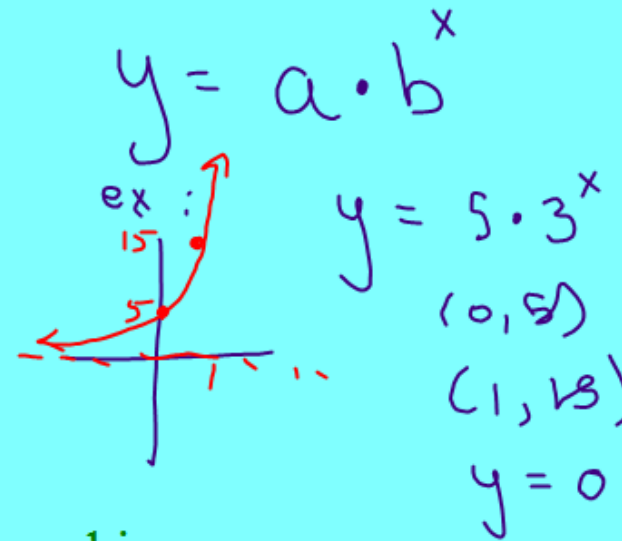
- If  $a = 1$ , the point will be  $(1, b)$

The asymptote will be the line  $y = 0$

If  $b > 1$ , it will represent exponential growth and increase

If  $0 < b < 1$ , it will represent exponential decay and decrease

So, to graph an exponential function, simply find your 2 points, draw your asymptote, and draw your growth or decay function.



Of course this will all be a little different when we introduce transformations tomorrow!

**HW #2: Graphing exponential functions**