

Warmup:

Simplify.

$$1) \quad -2\sqrt{54} - 3\sqrt{45} + 3\sqrt{45}$$

$$-6\sqrt{6} - \cancel{9\sqrt{5}} + \cancel{9\sqrt{5}}$$

$$\underline{-6\sqrt{6}}$$

$$\begin{aligned} \sqrt{45} &= \sqrt{9 \cdot 5} \\ &= \sqrt{3 \cdot 3 \cdot 5} \\ &= 3 \cdot \sqrt{5} \end{aligned}$$

$$2) \quad 2\sqrt{20} - 3\sqrt{5} - 2\sqrt{12}$$

$$4\sqrt{5} - 3\sqrt{5} - 4\sqrt{3}$$

$$\begin{aligned} &1\sqrt{5} - 4\sqrt{3} \\ \text{or } &\sqrt{5} - 4\sqrt{3} \end{aligned}$$

Solving Radical Equations

A "radical" equation is an equation in which at least one variable expression is stuck inside a radical, usually a square root.

For example, this is a radical equation: $\sqrt{x} + 2 = 5$

.... but this is not: $x + \sqrt{2} = 5$

$$x = 5 - \sqrt{2}$$

$$x + 2 = 5$$

$$\cancel{(\sqrt{x})} + 2 = 5$$

$$x = 9$$



$$x = 9$$

$$\sqrt[3]{x} + 2 = 5$$

$$\cancel{(\sqrt[3]{x})} + 2 = 5$$

$$x = 27$$

The “radical” in “radical equations” can be any root, whether a square root, a cube root, or some other root.

The basic technique used to solve radical equations is raising both sides of an equation to some power.

We have to be careful when solving radical equations to watch out for extraneous solutions when we raise both sides of an equation to any even power.

For example, if I have $x = 1$.

Squaring both sides gives us $x^2 = (1)^2 = 1$, which has two solutions, 1 and -1. Clearly -1 is not a solution to the original equation. Therefore, we call this -1 an extraneous solution.

$$\begin{array}{r} x = 1 \\ +4 \quad +4 \end{array}$$

Steps to solving Radical Equations

1. Isolate a radical on one side of the equation
2. Raise each side of the equation to the smallest power that will eliminate the isolated radical
3. If any radicals remain in the equation from step 2, return to step 1 and continue the solution process
4. Solve the resulting equation to determine any possible solutions
5. Check all solutions to determine whether extraneous solutions may have resulted from step 2

Example 1: Solve the radical equation

$$\text{a) } (\sqrt{x+2})^2 = (3)^2$$

$$x+2 = 9$$

-2 -2

$$x = 7 \quad \checkmark$$

$$\sqrt{7+2} = 3$$

$$\sqrt{9} = 3$$

$$3 = 3$$

$$\text{b) } (\sqrt{x-5})^2 = (4)^2$$

$$x-5 = 16$$

$$x = 21$$

$$\sqrt{21-5} = 4$$

$$\sqrt{16} = 4$$

$$4 = 4 \quad \checkmark$$

$$\text{c) } (\sqrt{4x+5}) + 1 = 0$$

-1 -1

$$(\sqrt{4x+5})^2 = (-1)^2$$

$$4x+5 = 1$$

$$4x = -4$$

$$x = -1$$

$$\sqrt{4(-1)+5} + 1 = 0$$

$$\sqrt{1} + 1 = 0$$

$$1 + 1 \neq 0$$

$$\rightarrow -1 + 1 = 0$$

$$\sqrt{3 \cdot 2 - 2} + 2 = 0$$

$$(d) \sqrt{3x-2} + 2 = 0$$

$$\begin{array}{r} 3x - 2 = 4 \\ +2 \quad +2 \end{array}$$

$$\frac{3x}{3} = \frac{6}{3}$$

$$\sqrt{x=2}$$

$$\begin{array}{l} \overline{(-2+2=0)} \\ 2+2 \neq 0 \end{array}$$

$$\sqrt{4} + 2 = 0$$

$$e) \sqrt{x+3} = (x+1)^2$$

$$\begin{array}{r} x+3 = x^2+2x+1 \\ -x-3 \quad -x-3 \end{array}$$

$$0 = x^2 + x - 2$$

$$0 = (x-1)(x+2)$$

$$0 = x-1$$

$$x=1$$

$$\begin{array}{r} -2 \\ -1 \times 2 \\ 1 \end{array}$$

$$0 = x+2$$

$$x=-2$$

$$f) \sqrt{x-5} = (x-7)^2$$

$$\begin{array}{r} x-5 = x^2-14x+49 \\ -x+5 \quad -x+5 \end{array}$$

$$0 = x^2 - 15x + 54$$

$$0 = (x-6)(x-9)$$

$$0 = x-6$$

$$x=6$$

$$0 = x-9$$

$$x=9$$

$$g) \sqrt{7x+1} - 1 = 2x$$

$\begin{array}{cc} +1 & +1 \\ (2x+1)(2x+1) \end{array}$

$$h) \sqrt{5x+1} - 1 = 3x$$

$$(\sqrt{7x+1})^2 = (2x+1)^2$$

$$\begin{array}{r} 7x+1 \\ -7x-1 \\ \hline \end{array} = \begin{array}{r} 4x^2+4x+1 \\ -4x-1 \\ \hline \end{array}$$

$$0 = 4x^2 - 3x$$

$$x=0 \text{ \& } 3/4$$

Solving Radical Equations HW