

Factor the following trinomials:

$$x^2 + 11x + \underline{10}$$

$$x^2 - 7x - 30$$

$$x^2 - 19x + 48$$

$$(x+10)(x+1)$$

$$(x-10)(x+3)$$

$$(x-16)(x-3)$$

$$\begin{array}{l} 1 \cdot 10 \\ 2 \cdot 5 \\ \del{3 \cdot 7} \end{array}$$

$$\begin{array}{l} 1 \cdot 48 \\ 2 \cdot 24 \\ \boxed{3 \cdot 16} \\ 4 \cdot 12 \\ 6 \cdot 8 \end{array}$$

$$1) \underline{n^2} + 14n + 40$$
$$(n + 4)(n + 10)$$

$$a = 1$$
$$\underline{ax^2 + bx + c}$$

$$2) \underline{x^2} + 5x - 6$$
$$(x - 1)(x + 6)$$

$$3) \underline{n^2} + 8n + 15$$
$$(n + 5)(n + 3)$$

$$4) \underline{m^2} + 3m - 18$$
$$(m - 3)(m + 6)$$

$$5) \underline{m^2} - 5m - 6$$
$$(m + 1)(m - 6)$$

$$6) \underline{v^2} - 3v - 54$$
$$(v + 6)(v - 9)$$

7) $m^2 + 5m - 50$

$$(m + 10)(m - 5)$$

8) $p^2 + 11p + 18$

$$(p + 2)(p + 9)$$

9) $v^2 - 6v - 16$

$$(v - 8)(v + 2)$$

10) $a^2 + 16a + 63$

$$(a + 9)(a + 7)$$

11) $p^2 - 16p + 60$

$$(p - 6)(p - 10)$$

★ 12) $m^2 + 4m + 4$

$$(m + 2)^2$$

13) $a^2 + 12a + 35$

$(a + 5)(a + 7)$

14) $m^2 + 14m + 48$

$(m + 8)(m + 6)$

15) $b^2 + 8b + 7$

$(b + 1)(b + 7)$

16) $r^2 - 3r - 4$

$(r + 1)(r - 4)$

E.Q.:

How do we factor quadratic expressions that are not trinomials with a leading coefficient of 1?

Yesterday you factored simple quadratic trinomials with a leading coefficient of 1

$$\underline{x^2 - 7x - 30 = (x - 10)(x + 3)}$$

Today we will look at our special products, and trinomials with leading coefficient not equal to 1.

$$x^2 = 16$$

$$\begin{array}{ccc} & -16 & \\ 4 & \times & -4 \\ & 0 & \end{array}$$

$$(x+4)(x-4)$$

$$x^2 + \frac{14x}{\cancel{2}} + 49$$

Special Products:

$$x^2 + \frac{20x}{\cancel{2}} + 100$$

$$x^2 + 12x + 36$$

Perfect Squares

Difference of Squares

$$\boxed{a^2} + 2ab + \boxed{b^2}$$

$$x^2 + \frac{2 \cdot 5 \cdot x}{\cancel{2}} + 25$$

$$\underline{x^2 + 10x + 25}$$

Note:

Each of these can still be factored using the x-factor technique if the leading coefficient is 1

$$\underline{x^2} + 26x + \underline{25}$$

$$(x + 25)(x + 1)$$

$$\boxed{a^2} - \boxed{b^2}$$

$$x^2 - 16$$

$$x^2 - 9$$

$$x^2 - 4$$

$$\underline{9x^2} - \underline{25}$$

$$\frac{\cancel{N\cancel{D}T}}{x^2 - 8}$$

Perfect Squares

$$a^2 + 2ab + b^2$$

When we multiply out a perfect square binomial, our product will always follow this pattern.

Examples:

$$(x + 2)^2 = (x + 2)(x + 2) = \underline{x^2} + 4x + \underline{4}$$

$$\begin{array}{r} 4 \\ 2 \times 2 \\ \hline 4 \end{array}$$

$$(x - 3)^2 = (x - 3)(x - 3) = \underline{x^2} - \underline{6x} + \underline{9}$$

$$(2x - 4)^2 = (2x - 4)(2x - 4) = \underline{4x^2} - \underline{16x} + \underline{16}$$

$(2x - 4)^2$

Perfect Squares

$$a^2 + 2ab + b^2$$

Example:

$$x^2 - 18x + 81$$

$$(x - 9)^2$$

Is the 1st term a perfect square? \times

Is the 3rd term a perfect square? 9

Is the 2nd term twice the product of those squares? $2 \cdot 9 \cdot x$
 $18x$

Perfect Squares

$$a^2 + 2ab + b^2$$

Example:

$$\underline{25x^2} + \underline{20x} + \underline{4} = (5x + 2)^2$$

Is the 1st term a perfect square? $5x$

Is the 3rd term a perfect square? 2

Is the 2nd term twice the product of those squares?
 $2 \cdot 2 \cdot 5x$
 $20x$

Difference of Squares

$$a^2 - b^2$$

To factor a perfect square, we simply need to recognize this pattern.

Example:

$$\boxed{x^2 - 100}$$

$$\underline{\underline{(x+10)(x-10)}}$$

Is the 1st term a perfect square? *x*

Is the 2nd term a perfect square? *10*

Are those terms being subtracted? *Yes*

Difference of Squares

$$a^2 - b^2$$

1, 4, 9, 16, 25, 36, 49, 64, 81,

100, 121, 144, 169, 196, 225

Example:

$$\underline{x}^2 - \underline{1}$$

$$(x+1)(x-1)$$

Is the 1st term a perfect square? *x*

Is the 2nd term a perfect square? *1*

Are those terms being subtracted? *Yes*

Difference of Squares

$$a^2 - b^2$$

Sum of squares

$$x^2 + 9 \neq (x+3)(x+3)$$

$$x^2 + \underline{6x} + 9$$

Example:

$$\underline{16x^2} - \underline{25} = (4x+5)(4x-5)$$

Is the 1st term a perfect square? $4x$

Is the 2nd term a perfect square? 5

Are those terms being subtracted? Yes

Trinomials with a leading coefficient not equal to 1

Sometimes, we can factor out the leading coefficient:

Examples:

Factor out the 2!

$$\begin{array}{c} -6 \\ -6 \times +1 \\ -5 \end{array}$$

$$\underline{2}x^2 - \underline{10}x - \underline{12}$$

$$2(x^2 - 5x - 6)$$

$$2 \cdot (x-6)(x+1)$$

$$\underline{a}(x-r_1)(x-r_2)$$

Factor out the 3!

$$\underline{3}x^2 + \underline{33}x + \underline{30}$$

$$3(x^2 + 11x + 10)$$

$$3(x+10)(x+1)$$

F. O. I. L.

$$\underline{6x^2} + \underline{13x} + \underline{6}$$

$$(2x \quad 6)(3x \quad 1) \quad (2x \quad 3)(3x \quad 2)$$

$$(2x \quad 1)(3x \quad 6) \quad (3x \quad 2)(2x \quad 3)$$

$$(6x \quad 3)(x \quad 2)$$

$$(x \quad 6) \quad (x \quad 2)(6x \quad 3)$$

$$(6x \quad 1)(x \quad 6)$$

$$(6x \quad 6)(x \quad 1)$$

Trinomials with a leading coefficient not equal to 1

Sometimes, we **can not** factor out the leading coefficient:

Examples:

$$6x^2 + 13x + 6$$

First, multiply the leading coefficient and the constant term

$$6 \text{ times } 6 = 36$$

Trinomials with a leading coefficient not equal to 1

Sometimes, we **can not** factor out the leading coefficient:

$$6x^2 + 13x + 6$$

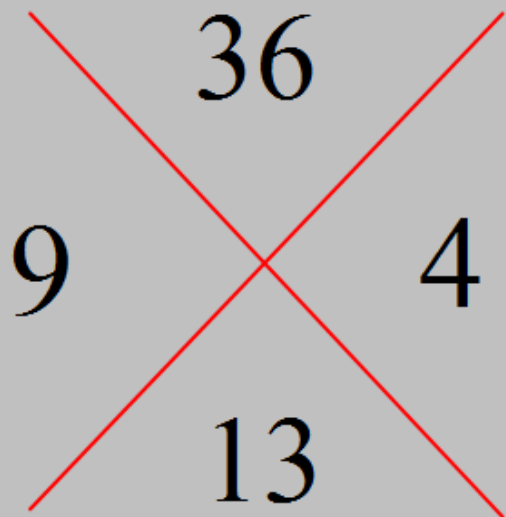
~~36~~

~~13~~

Second, set up your x-factor with this product on the top

Trinomials with a leading coefficient not equal to 1

Sometimes, we can not factor out the leading coefficient:



$$6x^2 + 13x + 6$$

Third, rewrite the middle term using these two numbers

$$6x^2 + \underline{9x} + \underline{4x} + 6$$

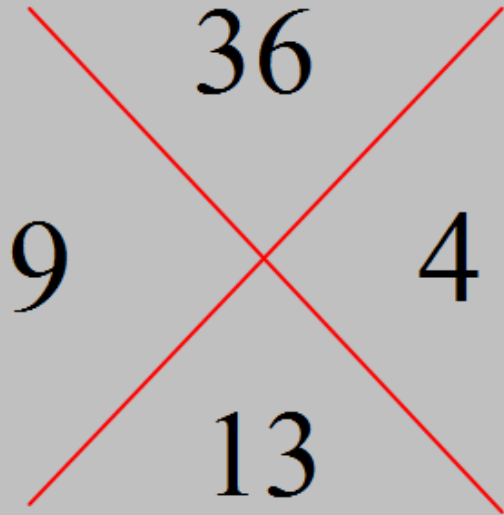
$$\underline{3x} (\underline{2x+3}) + \underline{2} (\underline{2x+3})$$

$$(2x+3)(3x+2)$$

Trinomials with a leading coefficient not equal to 1

Sometimes, we can not factor out the leading coefficient:

$$6x^2 + 13x + 6$$



Now we factor the first pair
and the second pair separately

$$6x^2 + 9x + 4x + 6$$

Trinomials with a leading coefficient not equal to 1

Sometimes, we **can not** factor out the leading coefficient:

~~36~~

~~9~~

~~4~~

13

$$\underline{6}x^2 + 13x + \underline{6}$$

$$\underline{6}x^2 + \underline{9x} + \underline{4x} + 6$$

$$\underline{3x}(\underline{2x + 3}) + \underline{2}(\underline{2x + 3})$$

Finally, we write our factored form

$$(\underline{3x + 2})(\underline{2x + 3})$$

Factoring by grouping

$$ax^2 + bx + c$$

1. Multiply a and c

ac

2. Set up the x-factor with ac on top and b on bottom

$$\begin{array}{c} \text{ac} \\ \times \\ \text{b} \end{array}$$

3. Find the two numbers that multiply to get ac and add to get b

$$\begin{array}{c} \text{ac} \\ \times \\ \text{d} \quad \text{e} \\ \text{b} \end{array}$$

4. Rewrite your middle term using these numbers

$$ax^2 + dx + ex + c$$

5. Factor the 1st two terms and the 2nd two terms separately

6. Write answer in factored form

Factoring by grouping

Examples:

$$10x^2 + 33x - 7$$

a.c

$$\begin{array}{r} -70 \\ 35 \quad -2 \\ \hline 33 \end{array}$$

$$\begin{array}{l} F \quad O \quad I \quad L \\ 10x^2 + 35x \quad \left\{ \begin{array}{l} -2x \\ -7 \end{array} \right. \\ 5x(2x+7) \quad \left\{ \begin{array}{l} -1 \\ (2x+7) \end{array} \right. \end{array}$$

$$\begin{array}{l} 1 \cdot 70 \\ 2 \cdot 35 \\ 5 \cdot 14 \\ 7 \cdot 10 \\ \hline \end{array}$$

~~$$10 \cdot 7$$~~

$$(2x+7)(5x-1)$$

Factoring by grouping

Examples:

$$4x^2 - 4x - 3$$

$$\star \textcircled{1} \begin{array}{l} 4 \cdot -3 \\ -12 \end{array}$$

$$\begin{array}{cc} -6 & -12 \\ \swarrow & \searrow \\ \cancel{2} & \cancel{2} \\ \swarrow & \searrow \\ -4 & 2 \end{array}$$

$$\begin{array}{l} 4x^2 + 2x \quad \{-6x - 3 \\ 2x(2x+1) \quad -3(2x+1) \end{array}$$

$$\begin{array}{l} 1 \cdot 12 \\ \textcircled{2 \cdot 6} \\ 3 \cdot 4 \end{array}$$

$$\frac{-6x}{2x} = \textcircled{-3}$$

$$\rightarrow (2x+1)(2x-3)$$

$$\begin{array}{l} 4x^2 - 6x \quad \{ +2x - 3 \\ \frac{2x}{\uparrow} (2x - \underset{\uparrow}{3}) \quad \frac{+1}{\uparrow} (2x - 3) \\ \rightarrow (2x-3)(2x+1) \end{array}$$

Factoring by grouping

Examples:

$$6x^2 + 11x - 10$$

~~$$\begin{array}{r} -60 \\ 15 \quad -4 \\ 11 \end{array}$$~~

$$\begin{aligned} &6x^2 + 15x \quad \left\{ \begin{array}{l} -4x \\ -10 \end{array} \right. \\ &3x(2x+5) \quad \left\{ \begin{array}{l} -2(2x+5) \end{array} \right. \\ &\quad (2x+5)(3x-2) \end{aligned}$$

Factoring by grouping

Examples:

$$40x^2 + x - 6$$

HW #3 Factoring Quadratics