Factor the following trinomials:

$x^2 + 11x + 10$

$= (x+10)(x+1)$

$x^2 - 7x - 30$

$(x-10)(x+3)$

$x^2 - 19x + 48$

$(x-16)(x-3)$

$1 \cdot 48$

$2 \cdot 24$

$\boxed{3 \cdot 16}$

$4 \cdot 12$

$6 \cdot 8$

1) $n^2 + 14n + 40$

$(n+4)(n+10)$

2) $x^2 + 5x - 6$

$(x-1)(x+6)$

3) $n^2 + 8n + 15$

$(n+5)(n+3)$

4) $m^2 + 3m - 18$

$(m-3)(m+6)$

5) $m^2 - 5m - 6$

$(m+1)(m-6)$

6) $v^2 - 3v - 54$

$(v+6)(v-9)$

7) $m^2 + 5m - 50$

$(m+10)(m-5)$

$(m-5)(m+10)$

8) $p^2 + 11p + 18$

$(p+2)(p+9)$

9) $v^2 - 6v - 16$

$(v-8)(v+2)$

10) $a^2 + 16a + 63$

$(a+9)(a+7)$

11) $p^2 - 16p + 60$

$(p-6)(p-10)$

★ 12) $m^2 + 4m + 4$

$(m+2)^2$

perfect squares

13) $a^2 + 12a + 35$

$(a+5)(a+7)$

14) $m^2 + 14m + 48$

$(m+8)(m+6)$

15) $b^2 + 8b + 7$

$(b+1)(b+7)$

16) $r^2 - 3r - 4$

$(r+1)(r-4)$

$\boxed{(x-4)(x+4)}$

$\boxed{x^2 - 16}$

Difference of squares

-16

0

E.Q.:

How do we factor quadratic expressions that are not trinomials with a leading coefficient of 1?

Yesterday you factored simple quadratic trinomials with a leading coefficient of 1

$$x^2 - 7x - 30 = (x - 10)(x + 3)$$

Today we will look at our special products, and trinomials with leading coefficient not equal to 1.

$(x+3)^2 = (x+3)(x+3) = x^2 + \underbrace{3x + 3x}_{2 \cdot 3x} + 9$

$x^2 + 6x + 9$

$(x+7)^2 = x^2 + 14x + 49$

$(x+7)(x+7)$

7x

7x

$(x+10)^2 = x^2 + 20x + 100$

$(x-5)^2 = x^2 - 10x + 25$

$2 \cdot 5x$

$x^2 + 26x + 25 = (x + 25)(x + 1)$

$2 \cdot 5x$

$$x^2 - 36 = (x+6)(x-6)$$

$-6x$

$6x$

$$x^2 - 49 = (x+7)(x-7)$$

Special Products:

Perfect Squares

$$a^2 + 2ab + b^2$$

Difference of Squares

$$a^2 - b^2$$

Note:

Each of these can still be factored using the x-factor technique if the leading coefficient is 1

Perfect Squares

$$a^2 + 2ab + b^2$$

When we multiply out a perfect square binomial, our product will always follow this pattern.

Examples:

$$(x + 2)^2 = (x + 2)(x + 2) = 1x^2 + 4x + 4$$

$$(x - 3)^2 = (x - 3)(x - 3) = 1x^2 - 6x + 9$$

$2 \cdot -3x$

$$(2x - 4)^2 = (2x - 4)(2x - 4) = 4x^2 - 16x + 16$$

$2 \cdot -4 \cdot 2x$

Perfect Squares

$$a^2 + 2ab + b^2$$

To factor a perfect square, we simply need to recognize this pattern.

Example:

$$x^2 + 10x + 25 = (x+5)^2$$

Is the 1st term a perfect square? Yes, x

Is the 3rd term a perfect square? Yes, 5

Is the 2nd term twice the product of those squares? $2 \cdot 5 \cdot x$

Perfect Squares

$$a^2 + 2ab + b^2$$

$\sqrt{81} = -9$ or 9

Example: $x^2 - 18x + 81$ $= (x - 9)^2$

Is the 1st term a perfect square? Yes, x

Is the 3rd term a perfect square? Yes, -9

Is the 2nd term twice the product of those squares? $2 \cdot -9 \cdot x$

$-18x$

Perfect Squares

1, 4, 9, 16, 25, 36, 49, 64

81, 100, 121, 144, 169, 196, 225

$$a^2 + 2ab + b^2$$

Example: $25x^2 + 20x + 4$ $= (5x + 2)^2$

Is the 1st term a perfect square? Yes, $5x$

Is the 3rd term a perfect square? Yes, 2

Is the 2nd term twice the product of those squares? $2 \cdot 2 \cdot 5x$

$20x$

Difference of Squares

$$a^2 - b^2$$

When we multiply out two binomials that represent a difference of squares, our product will always follow this pattern.

Examples:

$$(x-2)(x+2) = x^2 - 2^2 = x^2 - 4$$

$$(x-11)(x+11) = x^2 - 11^2 = x^2 - 121$$

$$(3x-4)(3x+4) = (3x)^2 - 4^2 = 9x^2 - 16$$

Difference of Squares

$$a^2 - b^2$$

To factor a perfect square, we simply need to recognize this pattern.

Example: $x^2 - 100$ $= (x+10)(x-10)$

Is the 1st term a perfect square? x

Is the 2nd term a perfect square? 10

Are those terms being subtracted? Yes

Difference of Squares

$$a^2 - b^2$$

Example: $x^2 - 1$ $(x+1)(x-1)$

Is the 1st term a perfect square? x

Is the 2nd term a perfect square? 1

Are those terms being subtracted? Yes

Difference of Squares

$$a^2 - b^2$$

$9x^2 - 100$

$(3x - 10)(3x + 10)$

Example:

$$16x^2 - 25$$

$(4x - 5)(4x + 5)$

Is the 1st term a perfect square? $4x$

Is the 2nd term a perfect square? 5

Are those terms being subtracted? Yes

$x^2 + 4$ Not Factorable

$\cancel{(x+2)(x-2)}$

$x^2 - 2x + 2x - 4$

$\cancel{(x+2)(x+2)}$

$x^2 + \underline{\underline{4x}} + 4$

Trinomials with a leading coefficient not equal to 1

Sometimes, we can factor out the leading coefficient:

Examples:

Factor out the 2!

-6

$-6 \quad +1$

-5

$$2x^2 - 10x - 12$$

$$2(1 \cdot x^2 - 5x - 6)$$

$$\boxed{2(x-6)(x+1)}$$

$$a(x-r_1)(x-r_2)$$

Factor out the 3!

$$3x^2 + 33x + 30$$

$$3(x^2 + 11x + 10)$$

$$3(x+10)(x+1)$$

$$6x^2 + 13x + 6$$

$(2x + 3)(3x + 2)$	$(6x + 3)(1x + 2)$
$(2x + 2)(3x + 3)$	$(6x + 2)(1x + 3)$
$(2x + 1)(3x + 6)$	$(6x + 1)(1x + 6)$
$(2x + 6)(3x + 1)$	$(6x + 6)(1x + 1)$

$$x^2 + 11x + 10$$

$$(x + 10)(x + 1)$$

Trinomials with a leading coefficient not equal to 1

Sometimes, we **<u>can not</u>** factor out the leading coefficient:

Examples:

$$6x^2 + 13x + 6$$

First, multiply the leading coefficient and the constant term

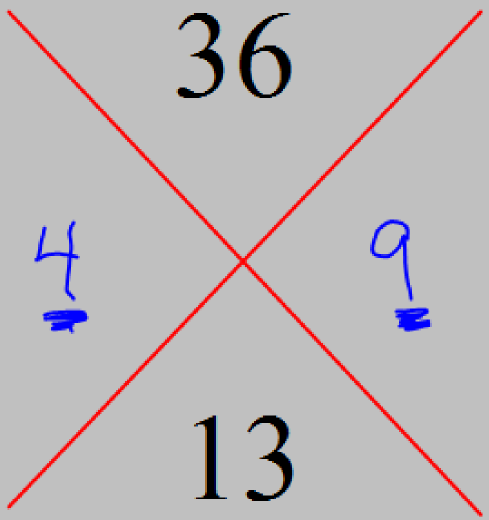
6 times 6 = 36

Trinomials with a leading coefficient not equal to 1

Sometimes, we **can not** factor out the leading coefficient:

$$6x^2 + 13x + 6$$

$1 \cdot 36$
$2 \cdot 18$
$3 \cdot 12$
$4 \cdot 9$
$6 \cdot 6$



Second, set up your x-factor with this product on the top

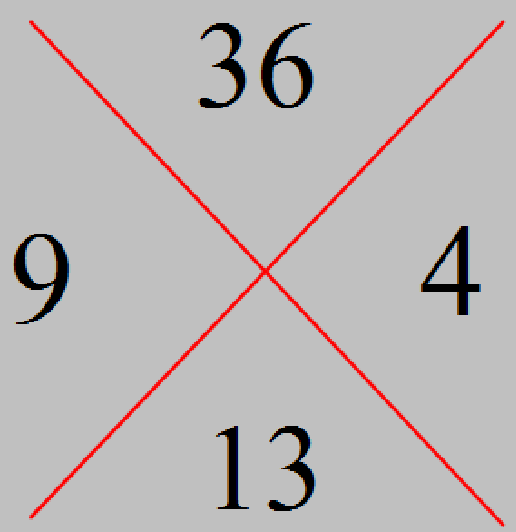
$$6x^2 + 4x + 9x + 6$$

F O I L

Trinomials with a leading coefficient not equal to 1

Sometimes, we **can not** factor out the leading coeff

$$6x^2 + 13x + 6$$



Third, rewrite the middle term using these two numbers

$$6x^2 + 9x + 4x + 6$$

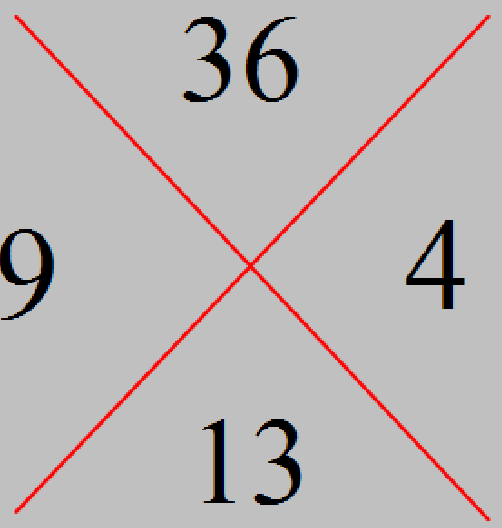
$\underline{3x}(\cancel{2x+3}) \; \underline{+2}(\cancel{2x+3})$

$(2x+3)(3x+2)$

Trinomials with a leading coefficient not equal to 1

Sometimes, we **<u>can not</u>** factor out the leading coefficient:

$$6x^2 + 13x + 6$$

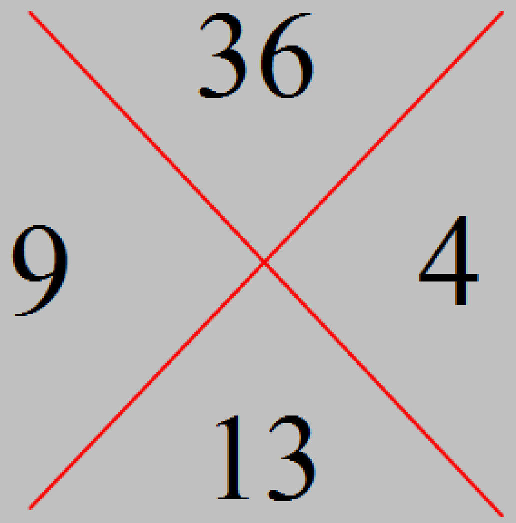


Now we factor the first pair
and the second pair separately

$$6x^2 + 9x + 4x + 6$$

Trinomials with a leading coefficient not equal to 1

Sometimes, we **<u>can not</u>** factor out the leading coefficient:



$$6x^2 + 13x + 6$$

$$6x^2 + 9x + 4x + 6$$

$$3x(2x + 3) + 2(2x + 3)$$

Finally, we write our factored form

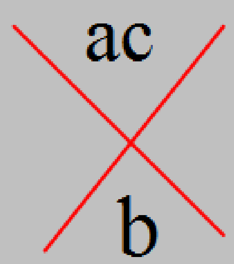
$$(3x + 2)(2x + 3)$$

Factoring by grouping

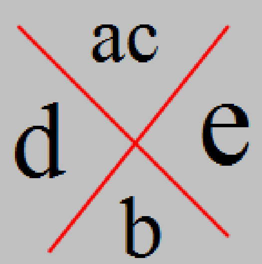
$$ax^2 + bx + c$$

1. Multiply a and c **ac**

2. Set up the x-factor with ac on top and b on bottom



3. Find the two numbers that multiply to get ac and add to get b



4. Rewrite your middle term using these numbers

$$ax^2 + dx + ex + c$$

5. Factor the 1st two terms and the 2nd two terms separately

6. Write answer in factored form

Factoring by grouping

Examples:

$$10x^2 + \cancel{33x} - 7$$

-70

-2 $+35$

33

$1 \cdot 70$

$2 \cdot 35$

$5 \cdot 14$

$7 \cdot 10$

$10x^2 - 2x + 35x - 7$

$2x(5x - 1) + 7(5x - 1)$

$(5x - 1)(2x + 7)$

$10x^2 + 35x - 2x - 7$

$5x(2x + 7) - 1(2x + 7)$

Factoring by grouping

Examples: $4x^2 - 4x - 3$

-12

$\boxed{-6 \quad +2}$

-4

$4x^2 - 6x + 2x - 3$

$2x(2x - 3) + 1(2x - 3)$

$(2x - 3)(2x + 1)$

Factoring by grouping

Examples:

$$6x^2 + 11x - 10$$

-60

$15 \qquad -4$

11

$$6x^2 + 15x - 4x - 10$$

$$3x(2x+5) - 2(2x+5)$$

$$(2x+5)(3x-2)$$

Factoring by grouping

Examples:

$$40x^2 + x - 6$$

HW #3 Factoring Quadratics