

Warmup:  
Graph the following exponential function.  
Be sure to label your two points and the asymptote.

$$y = a \cdot b^x$$

$$y = 12 \left( \frac{1}{3} \right)^x$$

asymptote:  $y = 0$

critical  
points

$$(0, a)$$

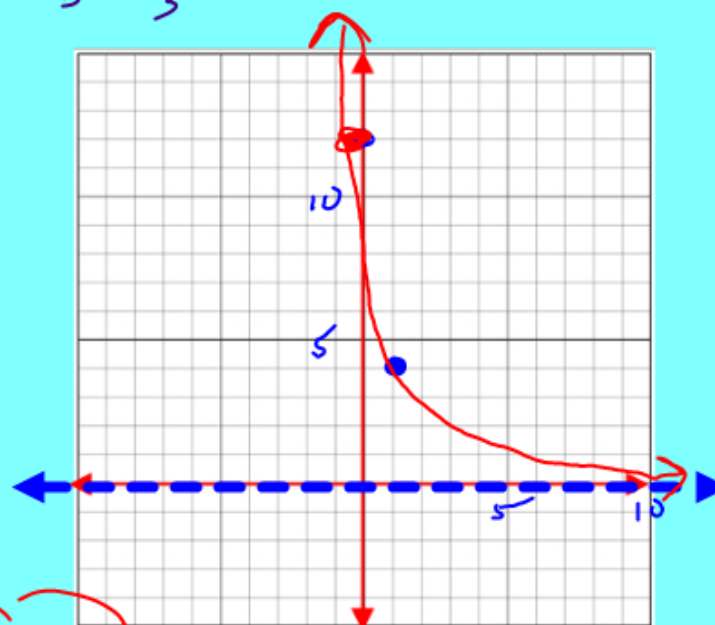
$$(0, 12)$$

$$(1, a \cdot b)$$

$$\left( 1, 12 \cdot \frac{1}{3} \right)$$

$$a = 12$$

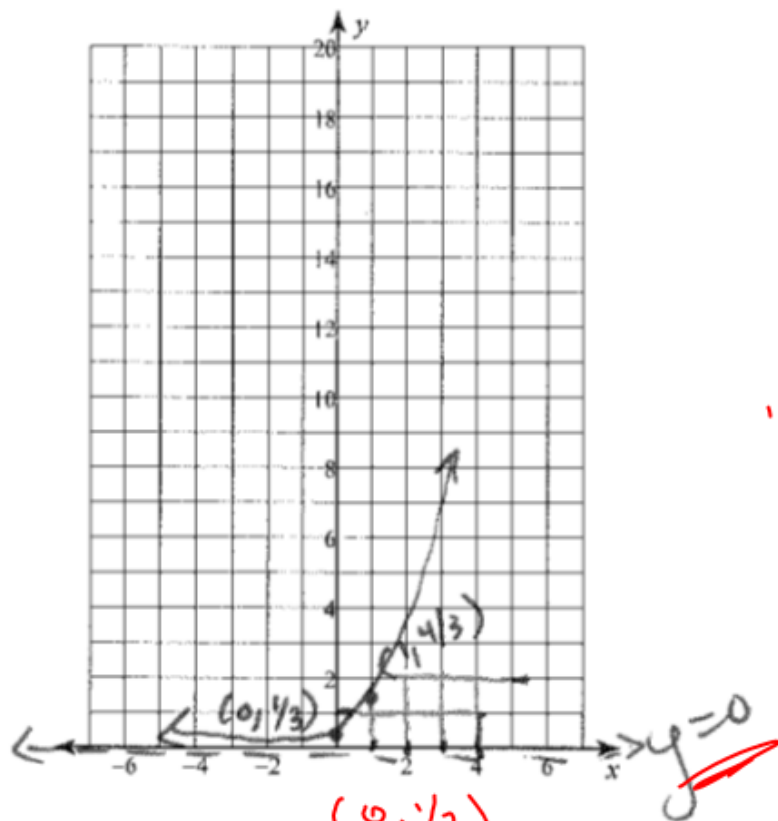
$$b = \frac{1}{3}$$



# HW #2: Graphing exponential functions

## Answer Key

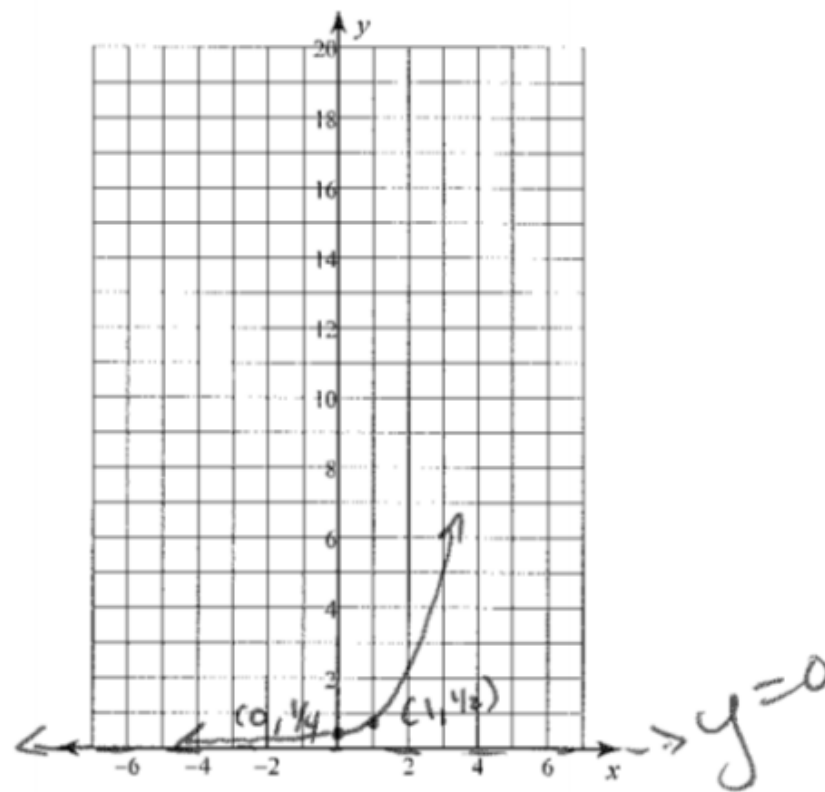
$$1) y = \frac{1}{3} \cdot 4^x$$



$$\star (0, 1/3)$$

$$(1, 4/3)$$

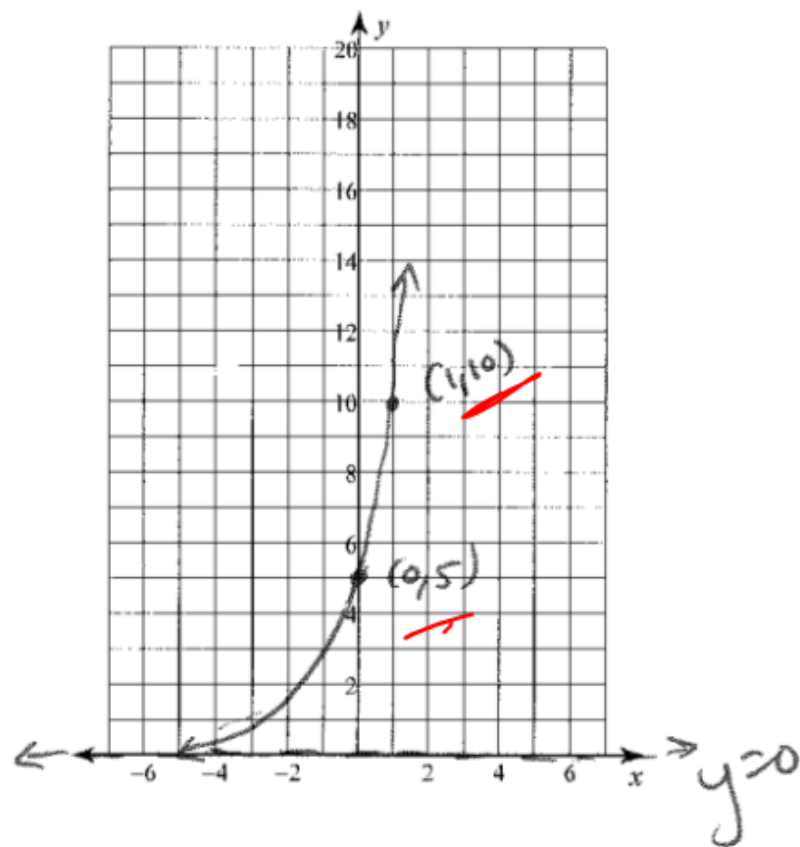
$$2) y = \frac{1}{4} \cdot 2^x$$



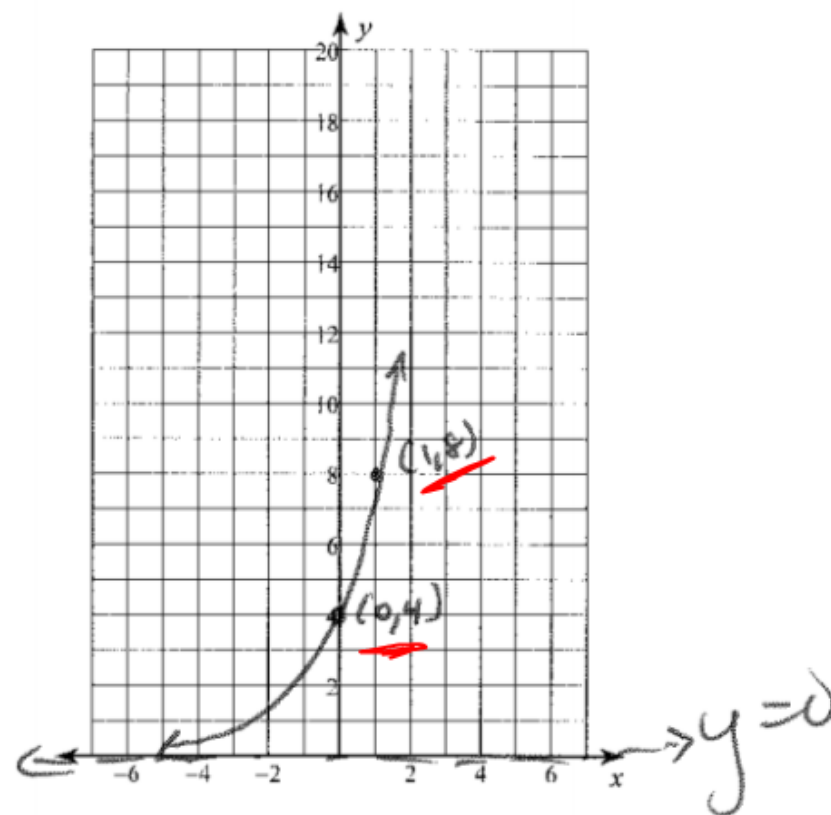
$$\star (0, 1/4)$$

$$(1, 1/2)$$

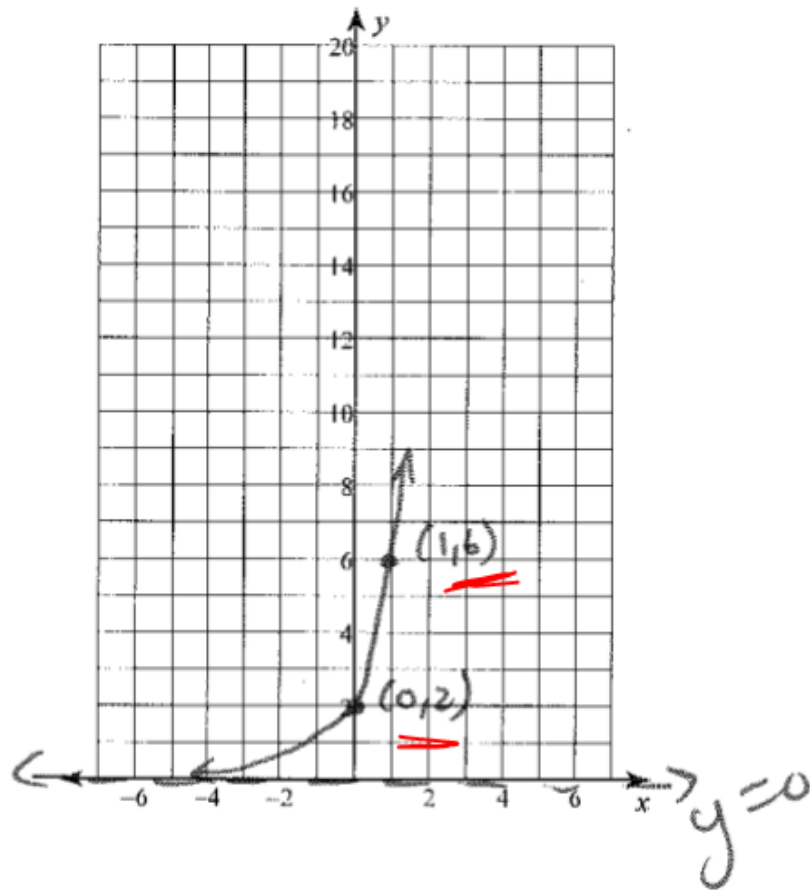
3)  $y = 5 \cdot 2^x$



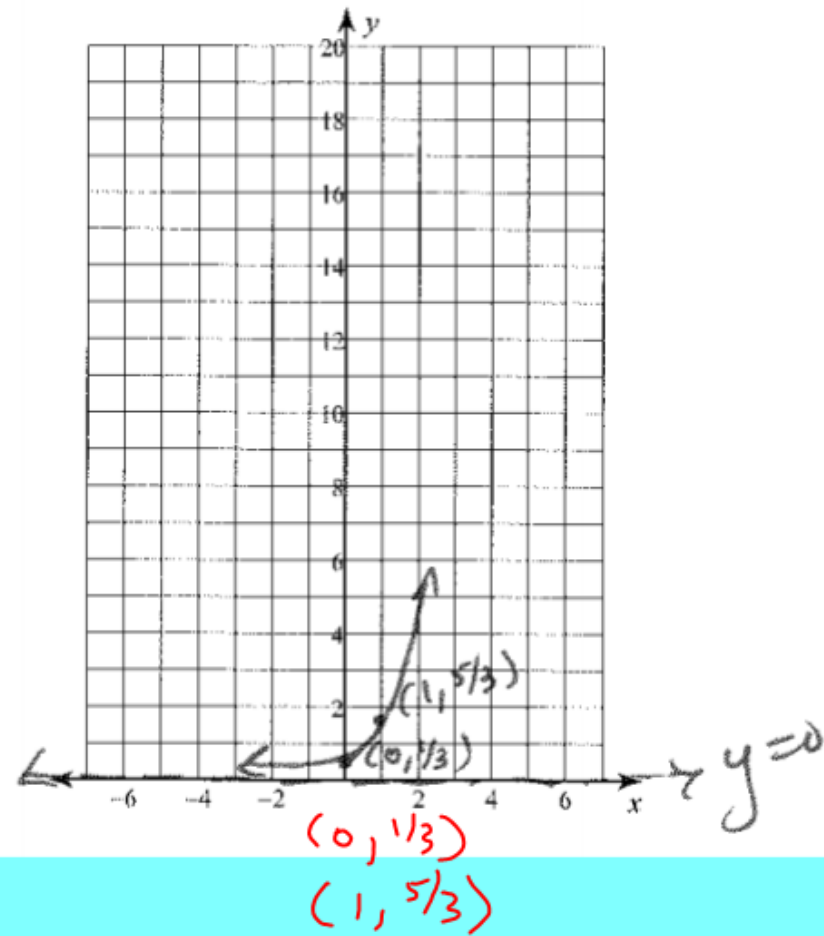
4)  $y = 4 \cdot 2^x$



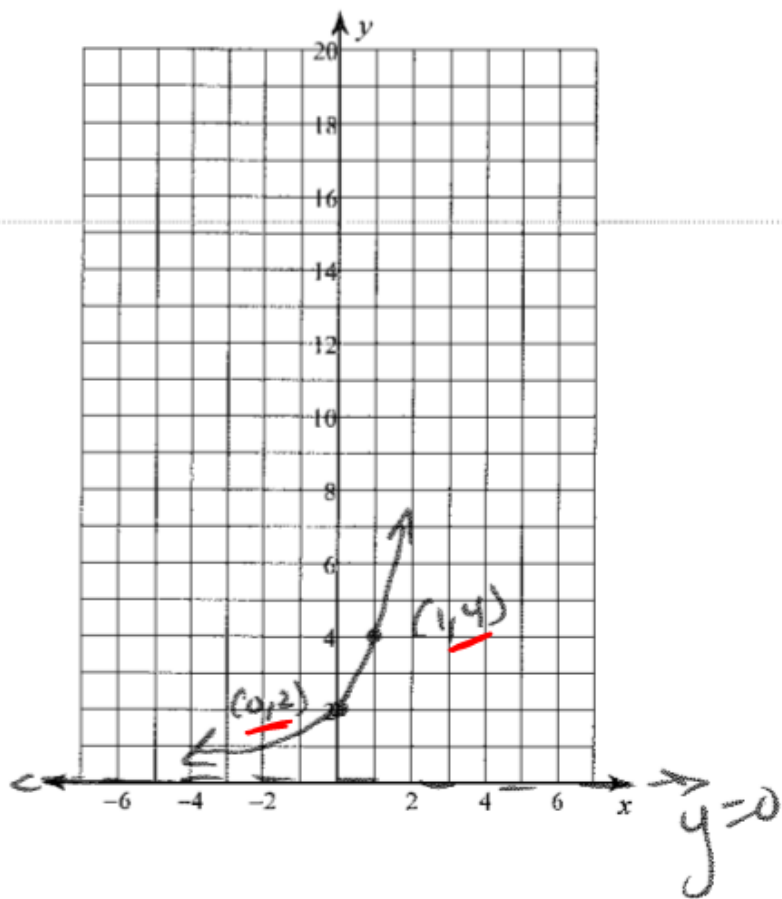
5)  $y = 2 \cdot 3^x$



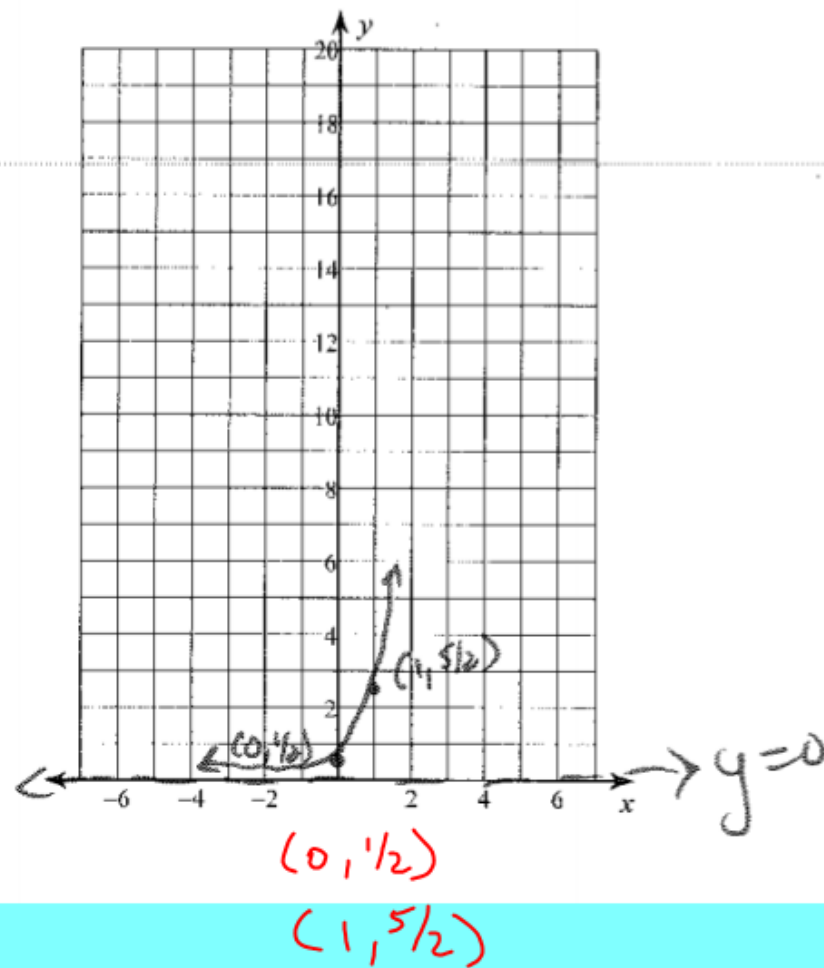
6)  $y = \frac{1}{3} \cdot 5^x$



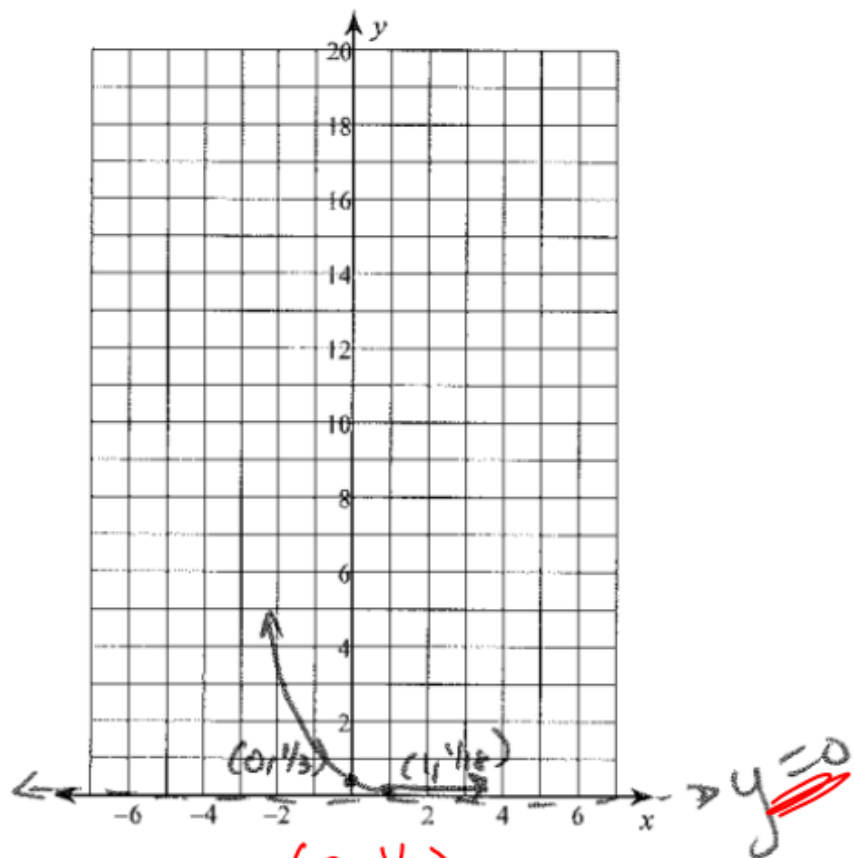
7)  $y = 2 \cdot 2^x$



8)  $y = \frac{1}{2} \cdot 5^x$



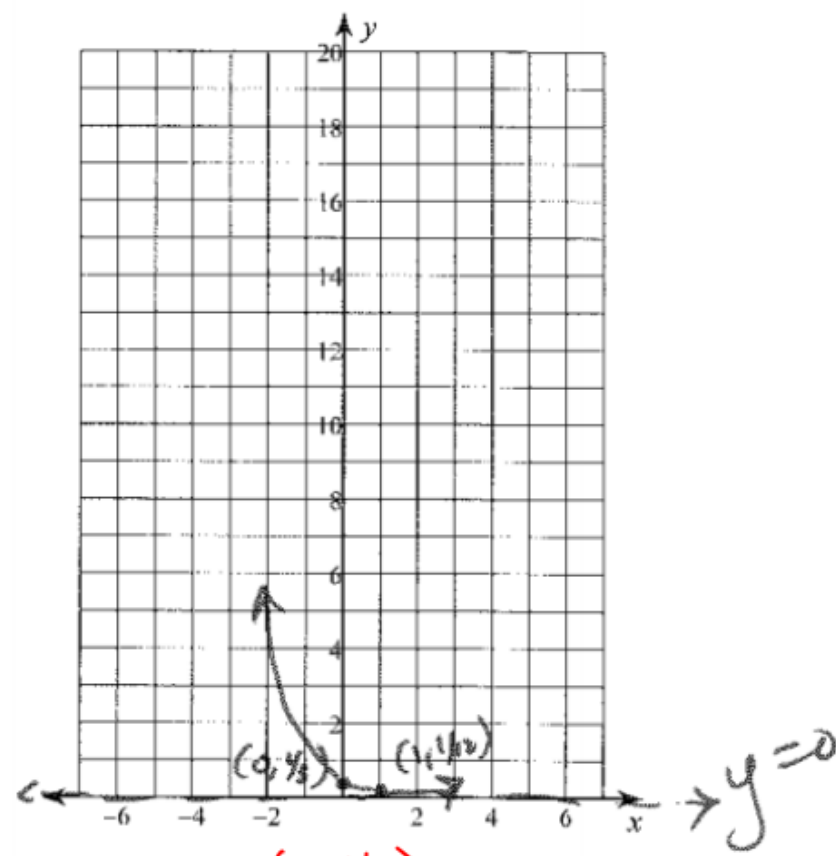
$$9) y = \frac{1}{3} \cdot \left(\frac{1}{6}\right)^x$$



$$\star (0, \frac{1}{3}) \star$$

$$(1, \frac{1}{18})$$

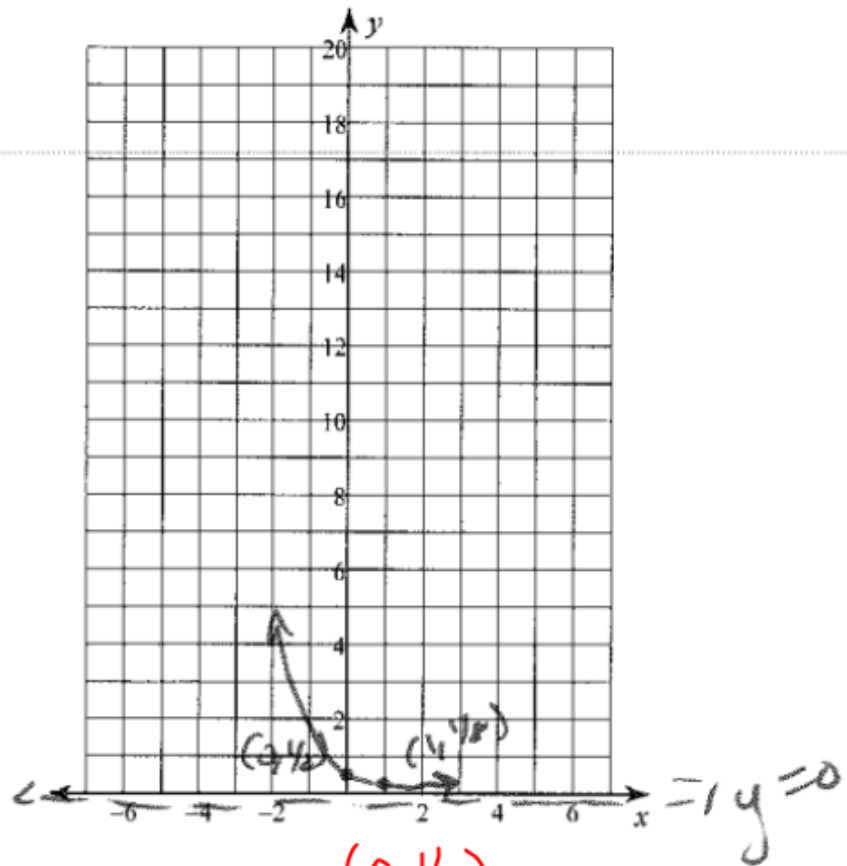
$$10) y = \frac{1}{3} \cdot \left(\frac{1}{4}\right)^x$$



$$(0, \frac{1}{3})$$

$$(1, \frac{1}{12})$$

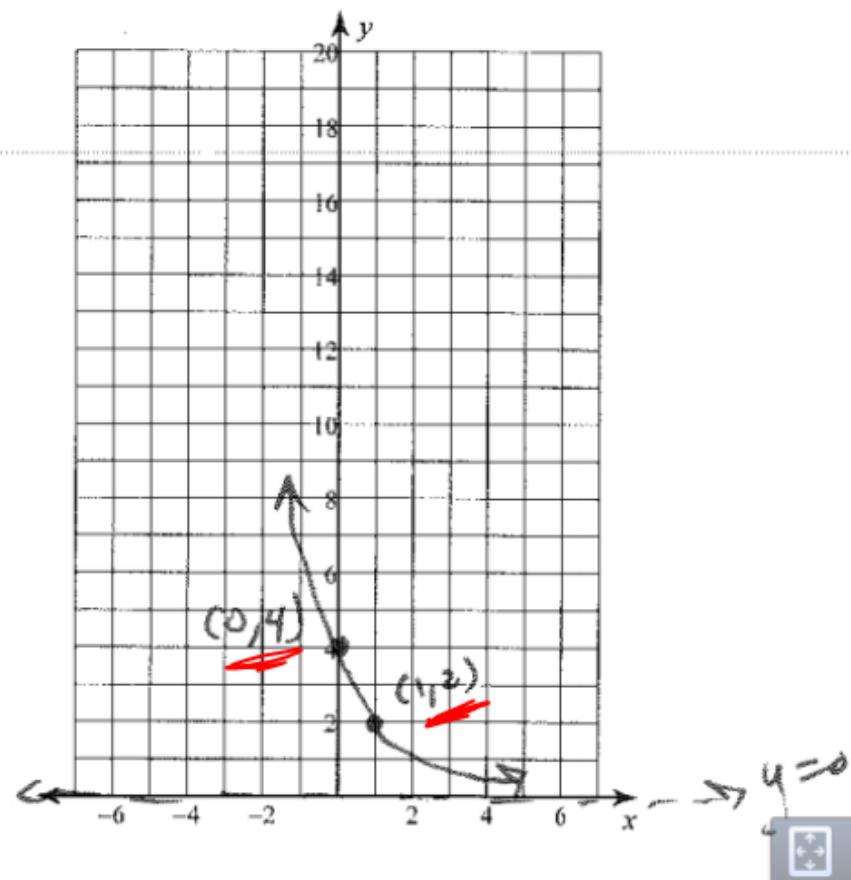
$$11) y = \frac{1}{2} \cdot \left(\frac{1}{4}\right)^x$$



$(-2, \frac{1}{2})$

$(1, \frac{1}{8})$

$$12) y = 4 \cdot \left(\frac{1}{2}\right)^x$$

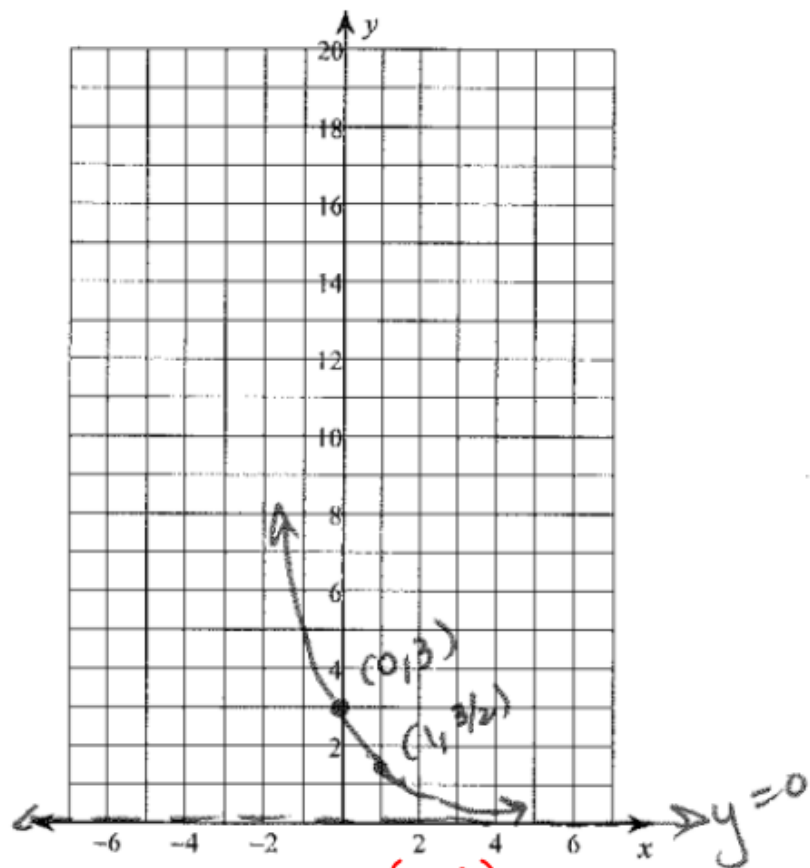


$(0, 4)$

$(1, 2)$



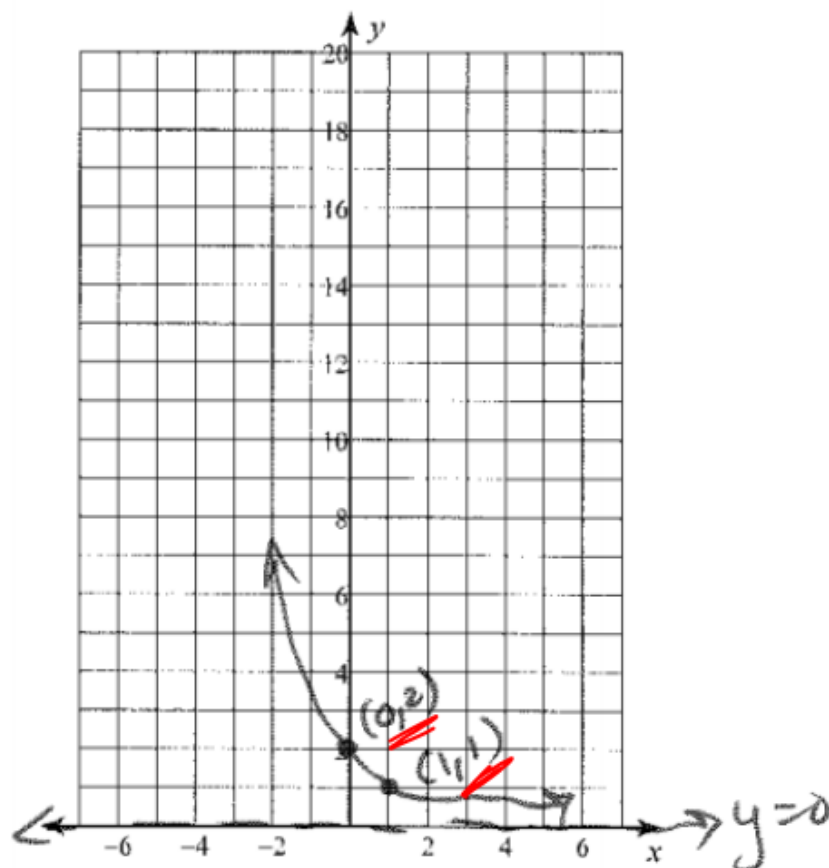
$$13) y = 3 \cdot \left(\frac{1}{2}\right)^x$$



$(0, 3)$

$(1, 3/2)$

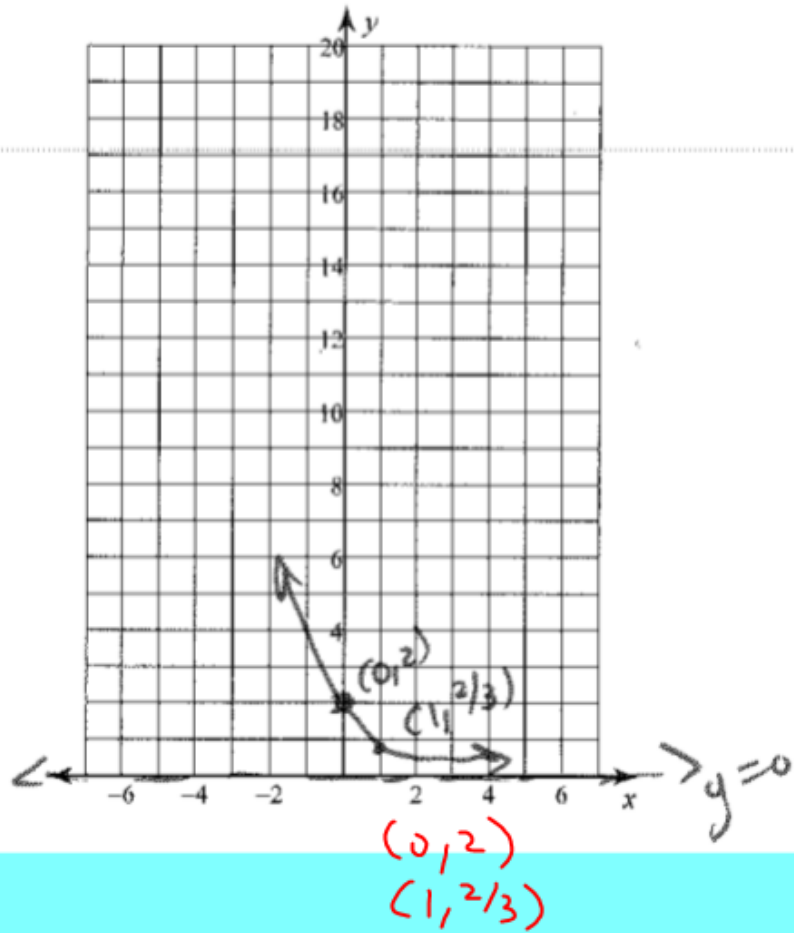
$$14) y = 2 \cdot \left(\frac{1}{2}\right)^x$$



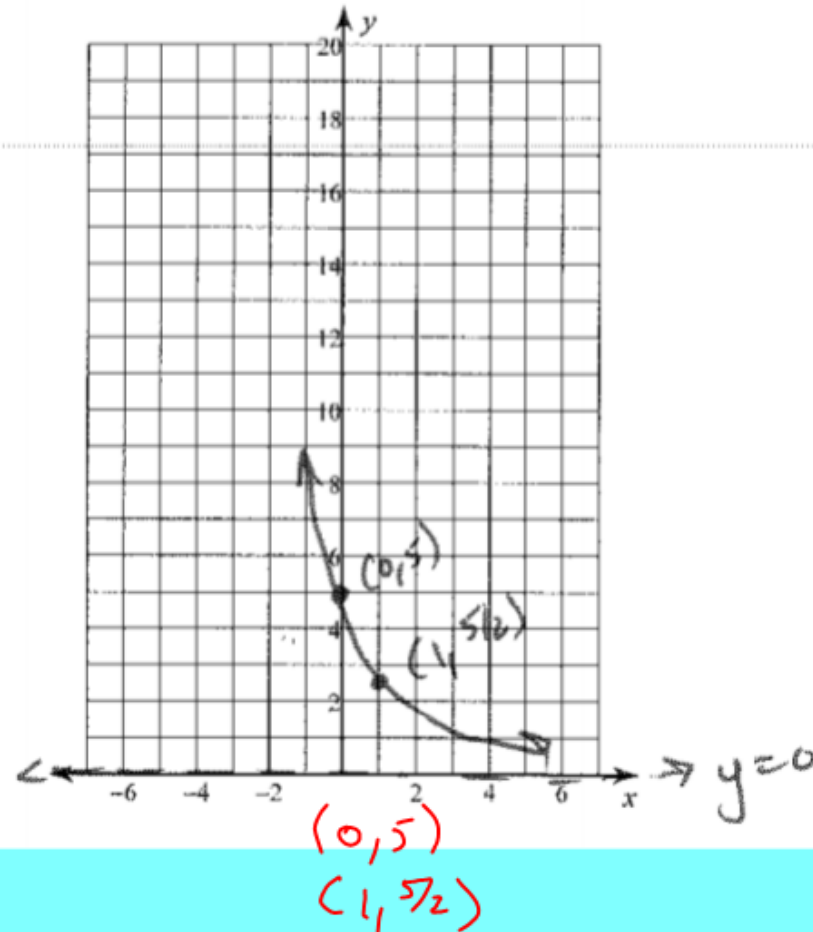
$(0, 2)$

$(1, 1)$

$$15) y = 2 \cdot \left(\frac{1}{3}\right)^x$$



$$16) y = 5 \cdot \left(\frac{1}{2}\right)^x$$



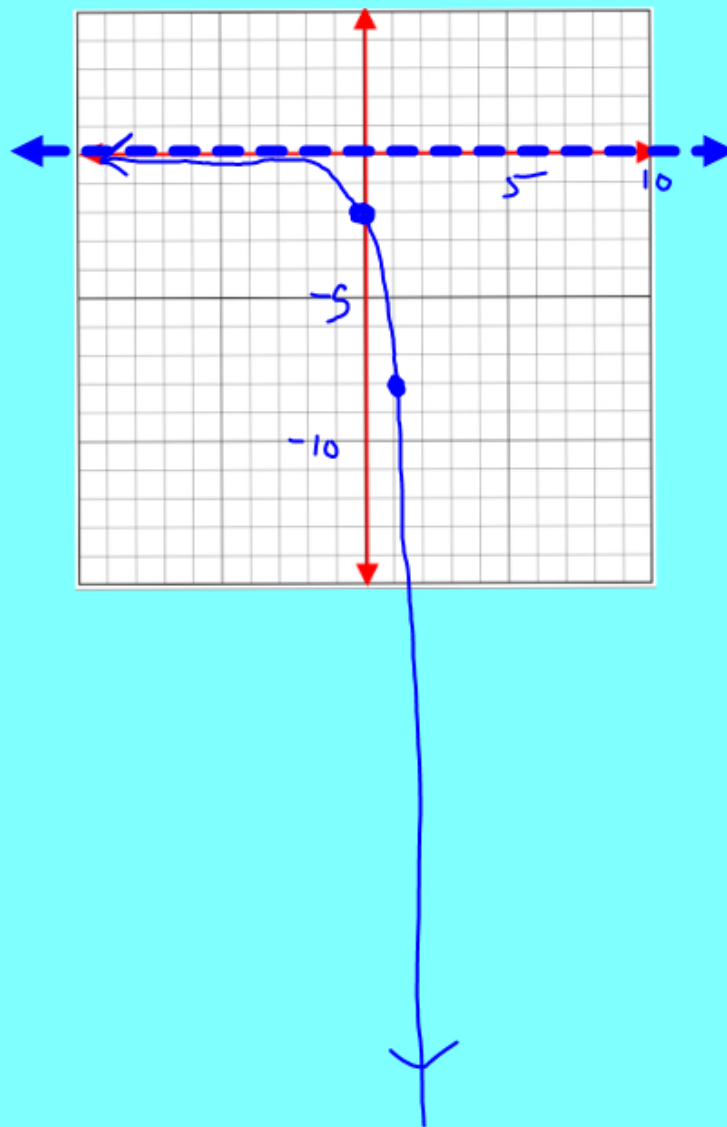
Warmup:  
Graph the following exponential function.  
Be sure to label your two points and the asymptote.

$$y = -2(4)^x$$

asymptote  $y = 0$

$(0, a)$   $(0, -2)$

$(1, ab)$   $(1, -8)$



$$y = 12\left(\frac{1}{3}\right)^x$$

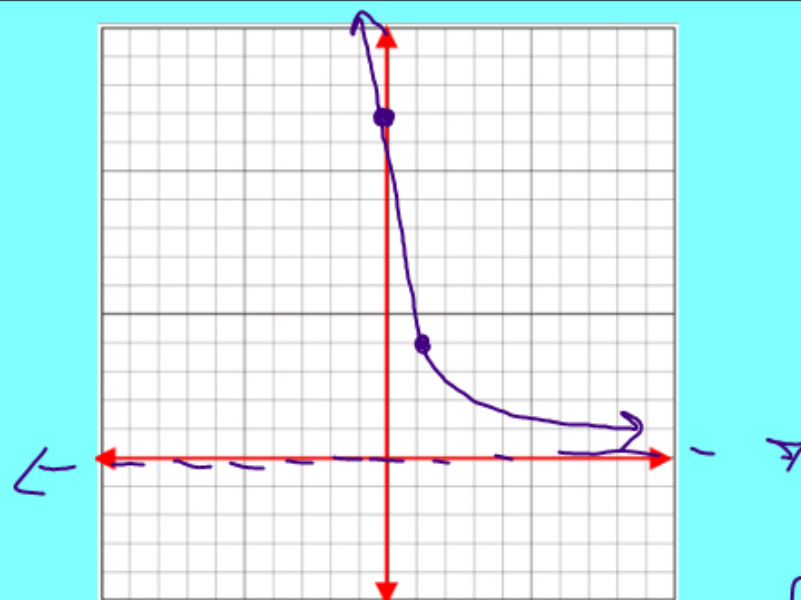
· Domain:  $(-\infty, \infty)$  \*

· Range:  $(0, \infty)$

· y-intercept:  $(0, 12)$

· Asymptote:  ~~$y = 0$~~

$$\begin{array}{l} (0, 12) \\ (2, 4/3) \end{array} \quad \frac{12 - 4/3}{0 - 2} = \left(\frac{-16}{3}\right)$$



· End Behavior: as  $x \rightarrow -\infty$   $y \rightarrow \infty$   
as  $x \rightarrow \infty$   $y \rightarrow 0$

\* ~~Increasing~~ or Decreasing?  
 $(-\infty, \infty)$

\* Slope Rate of Change from 0 to 2

# Characteristics of Exponential Function

## Function

Domain:  $(-\infty, \infty)$

Range:  $(\overset{\text{above}}{\text{asymptote}}, \infty)$  or  $(-\infty, \overset{\text{below}}{\text{asymptote}})$

★ y-intercept:  $(0, a)$  or plug in 0

♣ Asymptote:  $Y=0$  or  $y = k$

Increasing or Decreasing?

$(-\infty, \infty)$

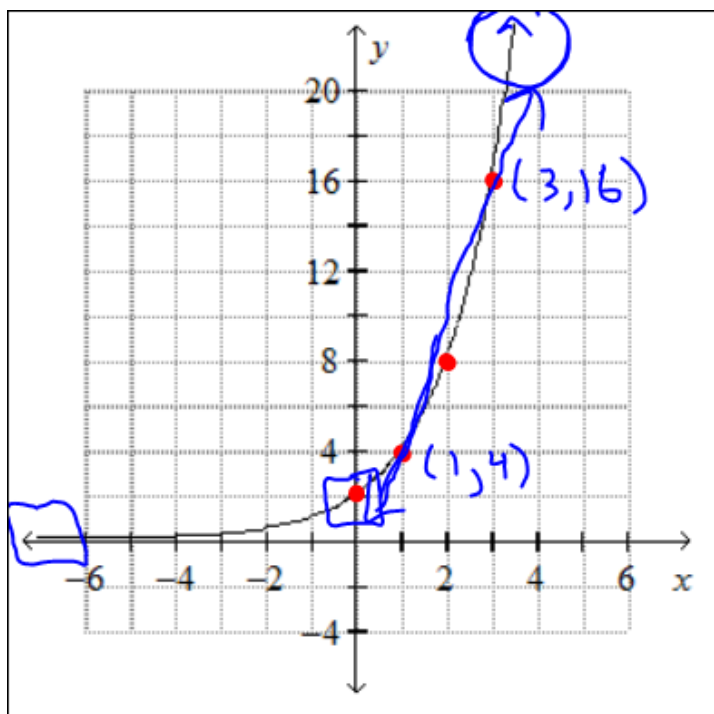
End Behavior:

$as\ x \rightarrow -\infty\ y \rightarrow \text{asymptote or } -\infty\ \text{or } \infty$

$as\ x \rightarrow \infty\ y \rightarrow \text{asymptote or } -\infty\ \text{or } \infty$

Rate of Change:

Find slope between two points



Domain:  $(-\infty, \infty)$

Range:  $(0, \infty)$

y-intercept:  $(0, 2)$

Asymptote:  $y = 0$

End Behavior:

as  $x \rightarrow -\infty$   $y \rightarrow 0$

as  $x \rightarrow \infty$   $y \rightarrow \infty$

$$\frac{16-4}{3-1} = \frac{12}{2} \Rightarrow \boxed{6}$$

Increasing or Decreasing?

$(-\infty, \infty)$

Rate of Change from 1 to 3:

$$y = a \cdot b^x$$

## Transformations of Exponential Functions:

Just like with the other parent functions, we can transform exponential functions. The basic form of transformations looks like:

$$f(x) = ab^{(x-h)} + k$$

Where: a stretch/shrink  
reflection  
 (a is neg.)    h horizontal shift    k vertical shift

Let's recall transformations of Quadratic Functions:

$$y = (x - 2)^2 - 4$$

Where is the vertex?  $(2, -4)$

What are the transformations from the parent function?

shift right 2  
down 4

opposite



Given the following exponential function:

$$f(x) = 3(2)^{x+3} - 2$$

What are the transformations of this function?

shifts left 3

shift down 2

$$y = 2(3)^{x-1} + 4$$

right 1  
up 4

$$y = 5(6)^x - 7$$


down 7

$$y = -6(5)^{x-5}$$

Right 5  
Reflect

## Graphing transformations of exponential functions:

To graph:

1. Identify the two critical points.  $(0, a)$   $(1, ab)$   
starts @
  2. Identify the asymptote (it's always  $y = 0$ ).
  3. Translate the critical points and asymptote.
  4. Draw a smooth curve. (only shift vertical)
- 

Identify your two critical points:  $(0, a)$   
 $\star (0, 3) \leftarrow (1, 6) \star$   $(1, ab)$

Identify your asymptote:

$\star y = 0 \star$

Identify the transformations:

Left 3 Down 2

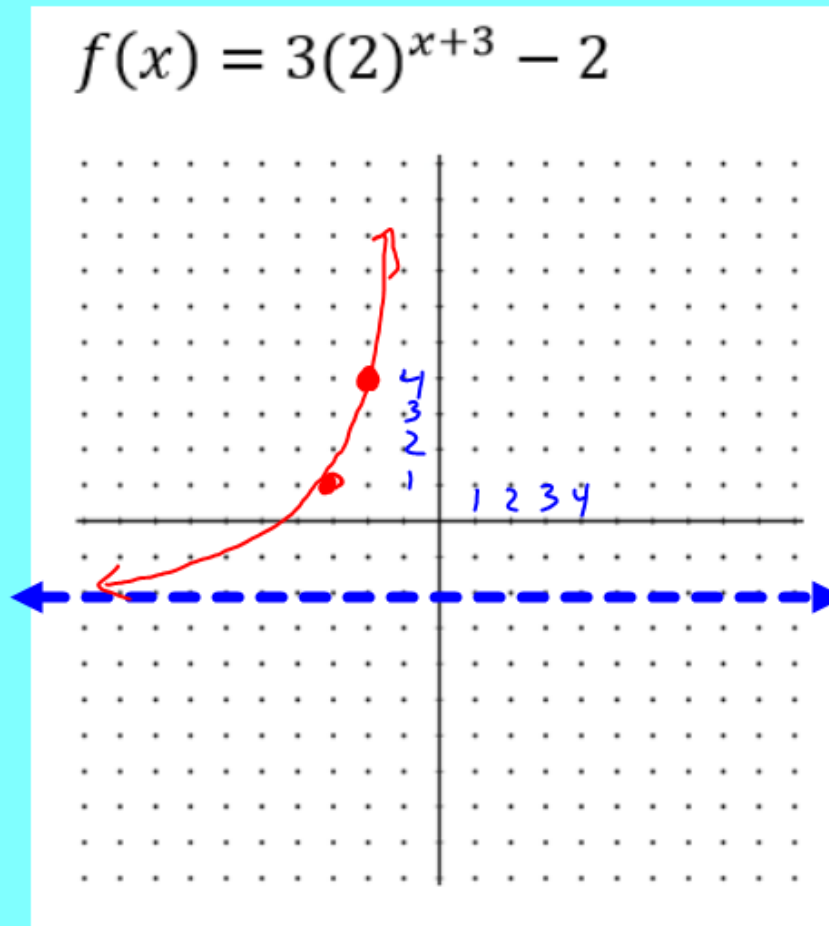
Where should the new critical points be located?  $(-3, 1)$   $(-2, 4)$

Where is the asymptote located now?

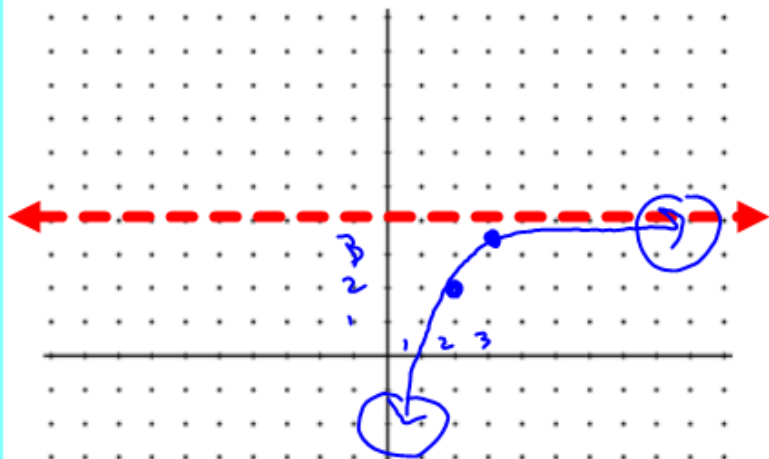
$y = -2$

GRAPH IT!!!

$$y = 3(2)^x$$



$$f(x) = -2\left(\frac{1}{4}\right)^{x-2} + 4$$



Start  
 $y=0$

$(0, -2) \rightarrow (2, 2)$

$(1, -1/2) \rightarrow (3, 3.5)$

Right 2 Up 4  
 $y=4$

a) y-intercept  $(0, -28)$

b) Increasing or Decreasing?  $(-\infty, \infty)$

c)  $a = -2$   $b = 1/4$

d) Critical Points:  $(2, 2)$  and  $(3, 3.5)$

e) Asymptote?  $y = 4$

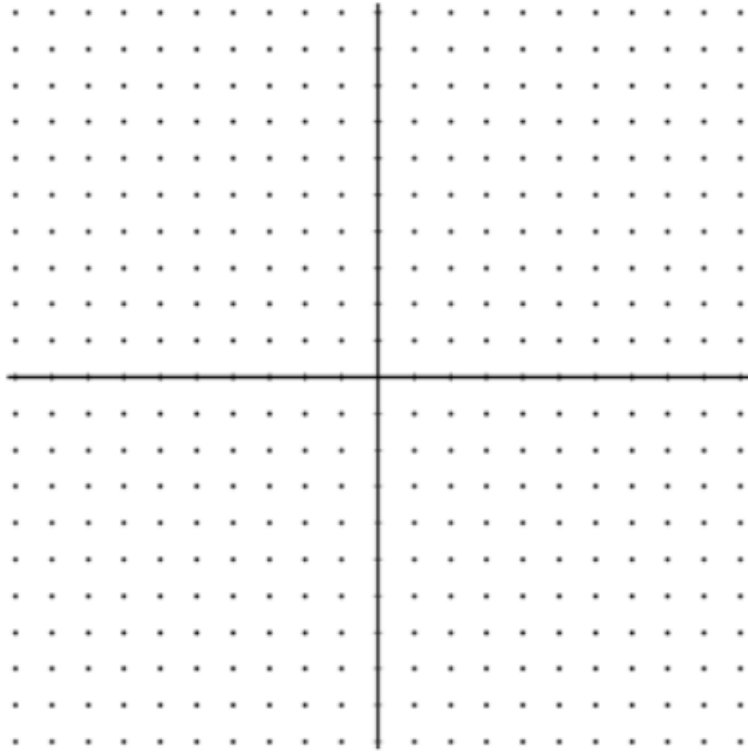
f) Domain  $(-\infty, \infty)$  Range  $(-\infty, 4)$

g) End Behavior \_\_\_\_\_

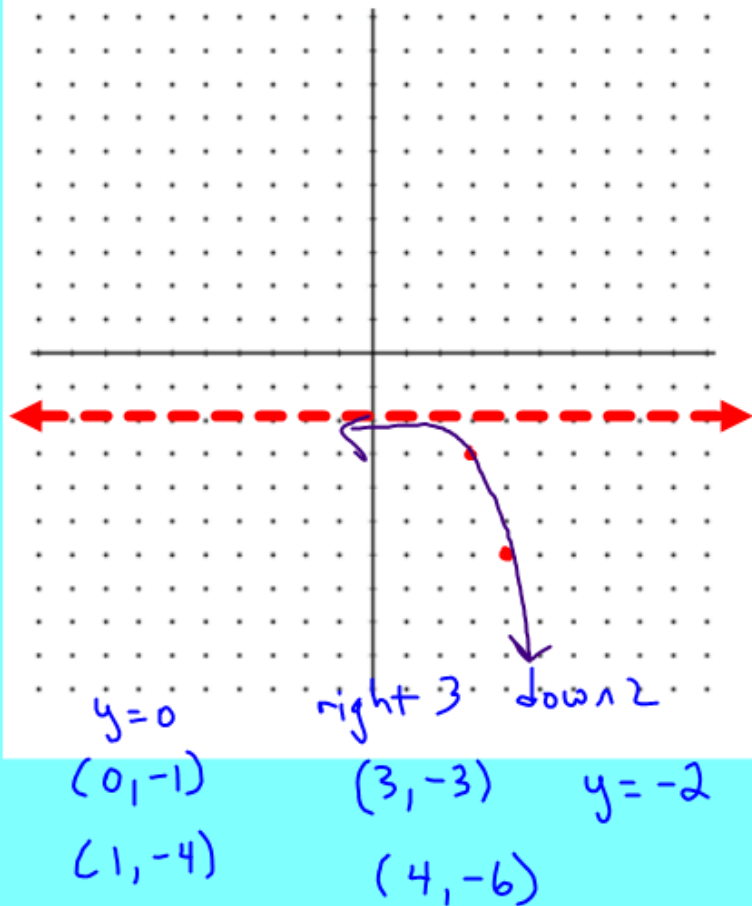
as  $x \rightarrow -\infty$   $y \rightarrow -\infty$

as  $x \rightarrow \infty$   $y \rightarrow 4$

$$f(x) = \frac{1}{2} \left(\frac{1}{2}\right)^x + 4$$



$$f(x) = -(4)^{x-3} - 2$$



a) y-intercept  $(0, \frac{-129}{64}) \approx (0, -2.02)$

b) Increasing or Decreasing?  $(-\infty, \infty)$

c) a = -1 b = 4

d) Critical Points:  $(3, -3)$  and  $(4, -6)$

e) Asymptote?  $y = -2$

f) Domain  $(-\infty, \infty)$  Range  $(-\infty, -2)$

g) End Behavior \_\_\_\_\_

as  $x \rightarrow -\infty$   $y \rightarrow -2$

as  $x \rightarrow \infty$   $y \rightarrow -\infty$

## HW #3

# Graphing Transformations of Exponential Functions