# Warmup:

$(3)^3 + 2(3) - 3 = 30$

**Consider the polynomial function $h(x) = x^3 + 2x - 3$.**

a. **Divide $h$ by $x - 3$.** $x - 3 = 0$, $x = 3$

$$\frac{h(x)}{x-3} = \frac{x^3 + 2x - 3}{x-3}$$

$$= (x^2 + 3x + 11) + \frac{30}{x-3}$$

b. **Find $h(3)$.**

$h(3) = 30$


Fund. Thm.

3 roots

3 real 0 imag
1 real 2 imag

2 real 1 imag.

0 real 3 imag.

Remainder Theorem

| |
|---|
| |

$x - 2 = 0$

$x = 2$

What is the remainder when $3x^2 - 5x + 6$ is divided by $x - 2$ ?

A 8

B -4

C 28

D 32

$3(2)^2 - 5(2) + 6$

| 2 | 3 | −5 | 6 |
|---|---|---|---|
| | | 6 | 2 |
| | 3 | 1 | 8 |

What is the remainder when $5x^2 + 3x - 7$ is divided by $x + 9$ ?

A 439

B 425

C 385

D 371

$-9 \mid 5 \quad 3 \quad -7$

$\quad\quad -45 \quad 378$

$5 \quad -42 \quad 371$

- or -

$5(-9)^2 + 3(-9) - 7$

What is the remainder when $2x^2 - 3x + 5$ is divided by $2x - 1$ ?

$2x - 1 = 0$

$x = .5$

A 2

B 3

C 4

D 5

K, A, L, A, M, E

.5 | 2 −3 5

1 −1

2 −2 4

# In Class Practice

# Remainder Theorem

# Factor Theorem

$f(x) = 2x^2 - 3x + 5$

$(2x - 1)$ $x = .5$

$f(.5) = 4$

**The Factor Theorem:**

When $f(c)=0$ then $x-c$ is a factor of the polynomial

*And the other way around, too:*

When $x-c$ is a factor of the polynomial then $f(c)=0$

5 a factor of 10?

5 a factor of 12?

$5 \times 2 = 10$

$\frac{10}{5} = 2$ R: 0

$\frac{12}{5} = 2$ R: 2

#1) $x^3 - 6x^2 + 11x - 6$ ; $x = 1$

$x - 1 = 0$

"$(x-1)$ is a factor."

$f(1) = 0$

1 | 1 | −6 | 11 | −6

| | 1 | −5 | 6 |
|---|---|---|---|
| 1 | 1 | −5 | 6 | 0 |

$(x^2 - 5x + 6)$

6, −3, −2, −5

$x^3 - 6x^2 + 11x - 6 = (x-1)(x^2 - 5x + 6)$

$f(x) = (x-1)\cdot(x-3)\cdot(x-2)$

$x = 1$ $x = 3$ $x = 2$

## Why Is This Useful?

Knowing that $x-c$ is a factor is the same as knowing that $c$ is a root (and vice versa).

The **factor "x−c"** and the **root "c"** are the same thing

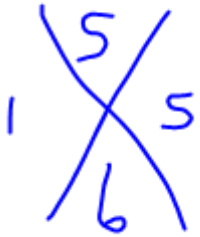
Know one and we know the other

$f(x) = x^3 + 5x^2 - x - 5$ = $(x-1)(x^2+6x+5)$

$x=1$

Is 1 a root of f(x)? yes.

| 1 | 1 | 5 | -1 | -5 |
|---|---|---|---|---|
| | 1 | 6 | 5 | |
| | 1 | 6 | 5 | 0 |




$(x-1)(x^2+6x+5)$

$(x-1)(x+1)(x+5)$

$x=1$ $x=-1$ $x=-5$

$f(x) = x^3 - 2x^2 - 20x - 24$ $= (x+2)(x^2 - 4x - 12)$

$(x+2)(x-6)(x+2)$

$x = -2$ $x = 6$ $x = -2$

Is (x + 2) a factor of f(x)?

$x = -2$ Yes

-2 | 1 -2 -20 -24

↓ -2 8 24

1 -4 -12 | 0

6 and -2 mult. of 2

$f(x) = 6x^3 + 17x^2 - 5x - 6$ $= (x+3)(6x^2 - x - 2)$

Is (x + 3) a factor of f(x)? Yes

$$\begin{array}{r|rrrr} -3 & 6 & 17 & -5 & -6 \\ & \downarrow & -18 & 3 & 6 \\ \hline & 6 & -1 & -2 & 0 \end{array}$$

$\rightarrow (x+3)(6x^2 - x - 2) \leftarrow$

$\Rightarrow (x+3)(3x-2)(2x+1) \Leftarrow$

$x = -3$

$3x - 2 = 0$
$+2 \quad +2$
$\frac{3x}{3} = \frac{2}{3}$
$x = \frac{2}{3}$

$2x + 1 = 0$
$\frac{2x}{2} = \frac{-1}{2}$
$x = -\frac{1}{2}$


$6x^2 - 4x + 3x - 2$

$2x(3x-2) + 1(3x-2)$

$(3x-2)(2x+1)$

Which of the following is a factor of $2x^3 - x^2 - 21x + 18$ ?

A $x - 1$

B $x - 2$

C $x - 3$

D $x - 4$

Which of the following is a root of $3x^4 + 6x^3 - 4x^2 - 6x + 4$ ?

A zero

B 1

C -1

D -2

Which of the following is a root of $x^5 - 2x^4 - 9x^3 + 17x^2 - x + 6$ ?

A -4

B -3

C 3

D 4

Which of the following is a factor of $6x^3 + 5x^2 - 2x - 1$ ?

A $x - 1$

B $2x - 1$

C $3x - 1$

D $4x - 1$

# HW #4: Factor Theorem