Warmup:

Rewrite each equation in exponential form.

1) $\log_3 27 = 3$

$3^3 = 27$

2) $\log_2 \frac{1}{64} = -6$

$2^{-6} = \frac{1}{64}$

$2^6 = 64$

$2^{-6} = \frac{1}{64}$

Properties of Logs

There are some properties of logs that will help us simplify logarithmic expressions.

Recall

Write $2^3 = 8$ in logarithmic form: $\log_2 8 = 3$

Write $\log_5 625$ in exponential form: $5^x = 625$

EXAMPLES:

$\log_2 2 = 1$ $\quad \log_2 4 =$

x	2	4	8	16	32	64
$\log_2 x$	1	2	3	4	5	6

a. $\log_2 (2 \bullet 4) = \underline{\ 3\ }$ $\qquad (\log_2 2) + (\log_2 4) = \underline{\ 3\ }$

$\log_2 (8) =$ $\qquad 1 + 2$ $\qquad x^2 \bullet x^4 = x^6$

b. $\log_2 (2 \bullet 8) = \underline{\ 4\ }$ $\qquad (\log_2 2) + (\log_2 8) = \underline{\ 4\ }$

$\log_2 (16)$ $\qquad 1 + 3$

c. $\log_2 (2 \bullet 16) = \underline{\ 5\ }$ $\qquad (\log_2 2) + (\log_2 16) = \underline{\ 5\ }$

$\log_2 (32) =$ $\qquad 1 + 4$

Product Property:

Recall: Product Property for exponents: $a^{m} \cdot a^{n} =$

a^{m+n}

There is an equivalent Product Property for logarithms: $log_b(mn) =$

$\log_b m + \log_b n$

EXAMPLE:

a. $\log_2(16/2) = \underline{3}$ 　 $(\log_2 16) - (\log_2 2) = \underline{3}$

$\log_2(8) =$ 　 $4 - 1$

b. $\log_2(64/32) = \underline{1}$ 　 $(\log_2 64) - (\log_2 32) = \underline{1}$

$\log_2(2) =$ 　 $6 - 5$

$$\frac{x^8}{x^5} = x^3$$

c. $\log_2(32/8) = \underline{2}$ 　 $(\log_2 32) - (\log_2 8) = \underline{2}$

$\log_2(4) =$ 　 $5 - 3$

d. $\log_2(8/4) = \underline{1}$ 　 $(\log_2 8) - (\log_2 4) = \underline{1}$

$\log_2(2) =$ 　 $3 - 2$

Quotient Property:

Recall: Quotient Property for exponents: $\frac{a^m}{a^n} = a^{m-n}$

There is an equivalent Quotient Property for logarithms: $log_b\left(\frac{m}{n}\right) =$

$\log_b m - \log_b n$

Product and Quotient Properties of Logarithms:

For $m > 0$, $n > 0$, $b > 0$, and $b \neq 1$

Product Property: $\log_b(mn) =$ ____________

Quotient Property: $\log_b(m/n) =$ ____________

Ex. Simplify.

a. $\log_3(3 \bullet 9) =$

$\log_3 3 + \log_3 9$

$1 + 2$

$\boxed{3}$

b. $\log_4(64/16) =$

$\log_4 64 - \log_4 16$

$3 - 2$

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Write each expression as a single logarithm. Then simplify, if possible.

a. $\log_3 10 - \log_3 5$

$\log_3\left(\frac{10}{5}\right)$

$\log_3 2$

b. $(\log_b u + \log_b v) - \log_b uw$

$\log_b(uv) - \log_b(uw)$

$\log_b\left(\frac{\cancel{u}v}{\cancel{u}w}\right)$

$\log_b\left(\frac{v}{w}\right)$

Ex. Write each expression as a single log. Simplify if possible.

a. $\log_8 12 - \log_8 4$

$\log_8\left(\frac{12}{4}\right) = \boxed{\log_8 3}$

b. $\log_z 2a - \log_z b + \log_z bc$

$\log_z\left(\frac{2a}{b}\right) + \log_z(bc)$

$\log_z\left(\frac{2a}{\cancel{b}} \cdot \cancel{b}c\right) = \log_z(2ac)$

c. $\log_4 18 - \log_4 6$

$\log_4\left(\frac{18}{6}\right)$

$\boxed{\log_4 3}$

d. $\log_b 4x - \log_b 3y + \log_b y$

$\log_b\left(\frac{4x}{3y}\right) + \log_b(y)$

$\log_b\left(\frac{4x}{3\cancel{y}} \cdot \cancel{y}\right)$

$\boxed{\log_b\left(\frac{4x}{3}\right)}$

Power Property:

Recall: Power Property for exponents: $(a^m)^n =$ $a^{m \cdot n}$

There is an equivalent Power Property for logarithms: $log_b m^p =$

$\log_b (m^p) = p \cdot \log_b (m)$

Power Property of Logs:

For $m > 0$, $b > 0$, and $b \neq 1$

$$\log_b m^p = p \bullet \log_b m$$

Ex. Evaluate

a. $\log_3 27^{100}$

$= 100 \cdot \log_3 27$

$= 100 \cdot 3$

$= 300$

b. $\log_4 16^5$

$5 \cdot \log_4 16$

$5 \cdot 2$

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expand:

$\log_7 (x^2 \cdot y^3 \cdot z^5)$

$\log_7 x^2 + \log_7 y^3 + \log_7 z^5$

$= 2\log_7 x + 3\log_7 y + 5\log_7 z$

$\log_5 (a^3 \cdot b^2)^4 = 4[\log_5 (a^3 b^2)]$

or

$\log_5 (a^{12} \cdot b^8)$

$\log_5 a^{12} + \log_5 b^8$

$12 \cdot \log_5 a + 8\log_5 b$

$4[\log_5 a^3 + \log_5 b^2]$

$4[3 \cdot \log_5 a + 2 \cdot \log_5 b]$

$12\log_5 a + 8\log_5 b$

Condense to a single logarithm:

$$4\log_3 x + 3\log_3 y = \log_3 (x^4 \cdot y^3)$$

$$\frac{\log_2 x}{2} + \frac{3\log_2 y}{2} = \frac{1}{2}\cdot\log_2 x + \frac{3}{2}\cdot\log_2 y$$

$$\log_2 x^{1/2} + \log_2 y^{3/2}$$

$$\log_2 (x^{1/2}\cdot y^{3/2}) = \log_2 \sqrt{x^1 y^3}$$

$$\sqrt{x} = x^{1/2}$$

$$\sqrt[3]{x} = x^{1/3}$$

$$\sqrt[4]{x} = x^{1/4}$$

Classwork:

Properties of Logs
Worksheet