E.Q.: How do we create and solve linear equations in two variables?

Standard: **MGSE9-12.A.CED.2**

Create linear equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

(The phrase "in two or more variables" refers to formulas like the compound interest formula, in which $A = P(1 + r/n)^{nt}$ has multiple variables.)

Vocabulary:

<u>linear equation</u>:

- an algebraic equation in which each term is either a constant or the product of a constant and (the first power of) a single variable.

4 or −3

$4x$ $-3x$ $\frac{1}{2}x$

Linear equations can have one or more variables.

slope - intercept form

$8 = 4x + 2$

$y = 4x + 2$

$y = 5x + 0$

$y = \underline{m}x + \underline{b}$

slope

y-intercept

$y = x + 2$

$y = 5$

independent variable: (x)

- It is a variable that stands alone and isn't changed by the other variables you are trying to measure.
- For example, someone's age might be an independent variable.

dependent variable: (y)

- A dependent variable is what you measure in the experiment and what is affected during the experiment.
- The dependent variable responds to the independent variable.
- It is called dependent because it "depends" on the independent variable.

dependent variable

independent variable

$$y = mx + b$$

slope

y-intercept

Slope and Rate of Change:

- The word slope (gradient, incline, pitch) is used to describe the measurement of the steepness of a straight line.
- The higher the slope, the steeper the line.
- The slope of a line is a rate of change.

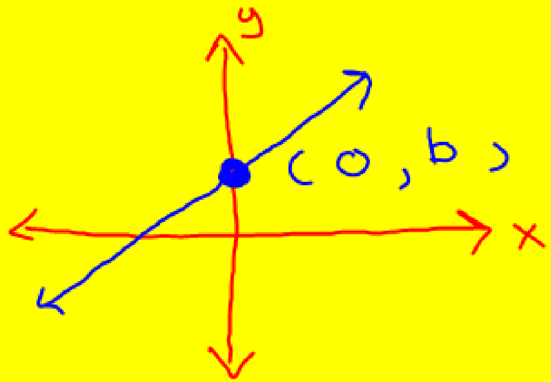
refers to a line

refers to slope.

y-intercept: (b)

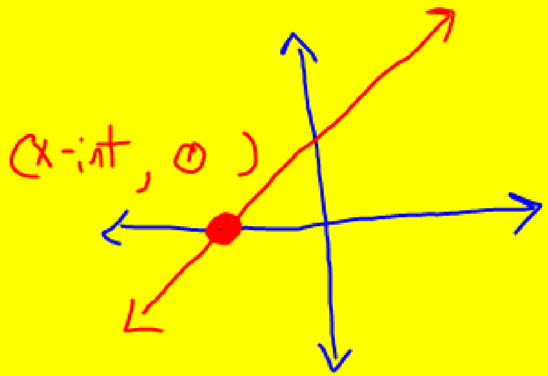
- is a point where the graph of an equation intersects with the y-axis of the coordinate system.
- these points satisfy x = 0.

Cartesian Plane



x-intercept:

- is a point where the graph of a function or relation intersects with the x-axis.
- these points satisfy y=0.



Multiple Representations of a Linear Relationship

$2 = \frac{2}{1} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x}$

Equation

$y = mx + b$

$y = 2x + 1$ y-int

slope-intercept form

slope

$y - 3 = 2(x - 1)$

point-slope form

$y - y_1 = m(x - x_1)$

Table

X	Y
-2	-3
-1	-1
0	1
1	3
2	5
3	7

run: +1, +1, +1

rise: +2, +2, +2

y-int

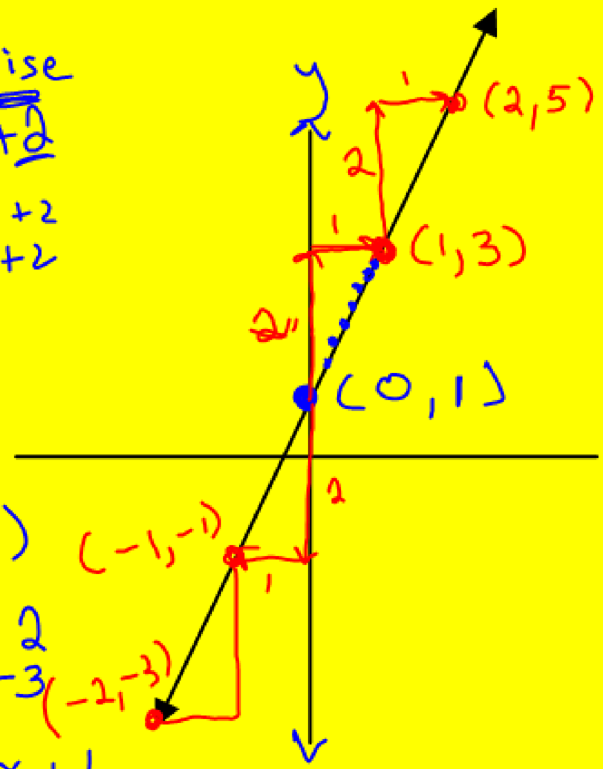
$y - 3 = 2(x - 1)$

$y - 3 = 2x - 2$

$+3 \qquad +3$

$y = 2x + 1$

Graph



Given the following equation, **y = 4x + 3**, create a table of values that is representative of that equation.

$$y = 4x + \underbrace{3}_{\text{y-int}}$$

slope

x	y
-2	4(-2) + 3 = -5
-1	4(-1) + 3 = -1
0	4(0) + 3 = 3
1	4(1) + 3 = 7
2	4(2) + 3 = 11
3	4(3) + 3 = 15

+1 +1 +1 +1 +1

+4 +4 +4 +4 +4

Given the following equation, **y = -2x + 5**, create a table of values that is representative of that equation.

slope

$y = -2x + \underbrace{5}_{\text{y-int}}$

X	y
-2	9
-1	7
0	5
1	3
2	1
3	-1

+1 +1 +1

-2 -2 -2

Given the following table,
is this table representative of a linear relationship?

If so, what is the equation?

$$y = \underbrace{m}_{\text{slope}}x + \underbrace{b}_{\text{y-int}}$$

x	y
-2	-15
-1	-10
0	-5
1	0
2	5
3	10

+1

+5, 5, 5, 5, 5

y-int

$$y = 5x - 5$$

$$y = 5(-2) - 5$$
$$y = -10 - 5$$
$$y = -15$$

$$y = 5(1) - 5$$
$$y = 5 - 5$$
$$y = 0$$

Given the following table,
is this table representative of a linear relationship?

If so, what is the equation?

x	y
-5	-15
0	-10
5	-5
10	0
15	5
20	10

+5 +5

5 5 5 5 5

$y = 1 \cdot x - 10$

$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{5}{5} = 1$

Given the following table,
is this table representative of a linear relationship? No

There is a ~~not~~ a constant rate of change!

If so, what is the equation?

x	y
-2	-15
0	-10
3	-5
10	0
15	5
50	10

+2 +5

+3 +5

slope = $\frac{5}{2}$ = 2.5

slope = $\frac{5}{3}$ = $1.\overline{6}$

Ex. The data below represent the value of a car as it depreciates over a period of 5 years.

+1

Years	0	1	2	3	4	5
Value	15000	12400	9800	7200	4600	2000

−2600

$\frac{-2600}{1} = -2600$

independent variable = years (x)

dependent variable = value (y)

slope = −2600

Calculate and interpret the slope. [rate of change]

For each additional year, the car's value decreased by \$2600.

Ex. The data below represent the value of a car as it depreciates over a period of 5 years.

Years	0	1	2	3	4	5
Value	15000	12400	9800	7200	4600	2000

State and interpret the y-intercept.

y-int: $15,000

When the car is new, at *time* = 0 years, the car is worth $15,000.

Ex. The data below represent the value of a car as it depreciates over a period of 5 years.

Years	0	1	2	3	4	5
Value	15000	12400	9800	7200	4600	2000

Write an equation that models this relationship:

$y = mx + b$

$y = -2600x + 15000$

Use your model to determine
the value of the car at 3.5 years.

"y" "x"

$$y = -2600x + 15000$$

$$y = -2600(3.5) + 15000$$

$$y = \$\ 5{,}900$$

Use your model to determine when "x" the car will be worth \$700? "y"

$y = -2600x + 15000$

$700 = -2600x + 15000$

$-15000 \qquad -15000$

$-14300 = -2600x$

$\frac{-14300}{-2600} = \frac{-2600x}{-2600}$

5.5 years = x

The table below shows the depth in meters of a scuba diver after a certain amount of time under water.

Position of Scuba Diver	
Time (s)	Depth (m)
x	y
0	−24
3	−18
6	−12
9	−6
12	0

independent variable =

dependent variable =

Find the average rate of change for this relationship. Interpret this value.

average rate of change = ____________________________________

For each additional _______________ that elapses, the diver has risen _______ meters under the surface of the water.

State and interpret the y-intercept. y-intercept = ___________

At time = ______, the scuba diver is __________ meters under the surface of the water.

Position of Scuba Diver	
Time (s)	Depth (m)
x	y
0	−24
3	−18
6	−12
9	−6
12	0

Write an equation that models this relationship:

$y = mx + b$

Position of Scuba Diver	
Time (s)	Depth (m)
x	y
0	−24
3	−18
6	−12
9	−6
12	0

Use your model to determine the diver's depth at 5 seconds.______

Use your model to predict how many seconds it takes the diver reach 9 meters below the surface. ______

Homework #4:
Two Variable
Equations