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# Monday, September 25th

— What do you call friends that love math? —  
Algebras

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## Warm-Up

Expand the following binomials.

$$(x+1)^2 \quad (x+1)^3 \quad (x+1)^4$$

# Review of Last Week

Combining like terms

Classifying polynomials

Adding and subtracting polynomials

Multiplying polynomials

$$(5x^2 + 3x - 6) - (3x^2 - x + 5)$$

This guy distributes into these!

$$= 5x^2 + 3x - 6 - 3x^2 + x - 5$$

$$= (5x^2 - 3x^2) + (3x + x) + (-6 - 5)$$

$$= 2x^2 + 4x - 11$$

# Review

$$(a + b)(c + d)$$

$$(a + b)(c + d) = ac + ad + bc + bd$$

F   O   I   L

$$(4x - 1)(3x + 7)$$

	4x	-1	
3x	12x <sup>2</sup>	-3x	= <u>12x<sup>2</sup> + 25x - 7</u>
+7	28x	-7	

# What is an exponent?

$$x^2$$

P E M D A S  
 $\searrow$   
 G

$$(x+1)^2$$

$$\begin{array}{l} 3(16) \\ 12x^2 \\ 19 \end{array}$$

$$3x^2$$

$$3(x)(x)$$

$$(3 \cdot 4)^2$$

What if...

$$(x+1)^3$$

$$(x+1)(x+1)(x+1)$$

$$(x^2+2x+1)(x+1)$$

$$x^3 + 1x^2 + 2x^2 + 2x + x + 1$$

$$x^3 + 3x^2 + 3x + 1$$

$$(x+1)^4$$

$$(x+1)^7$$

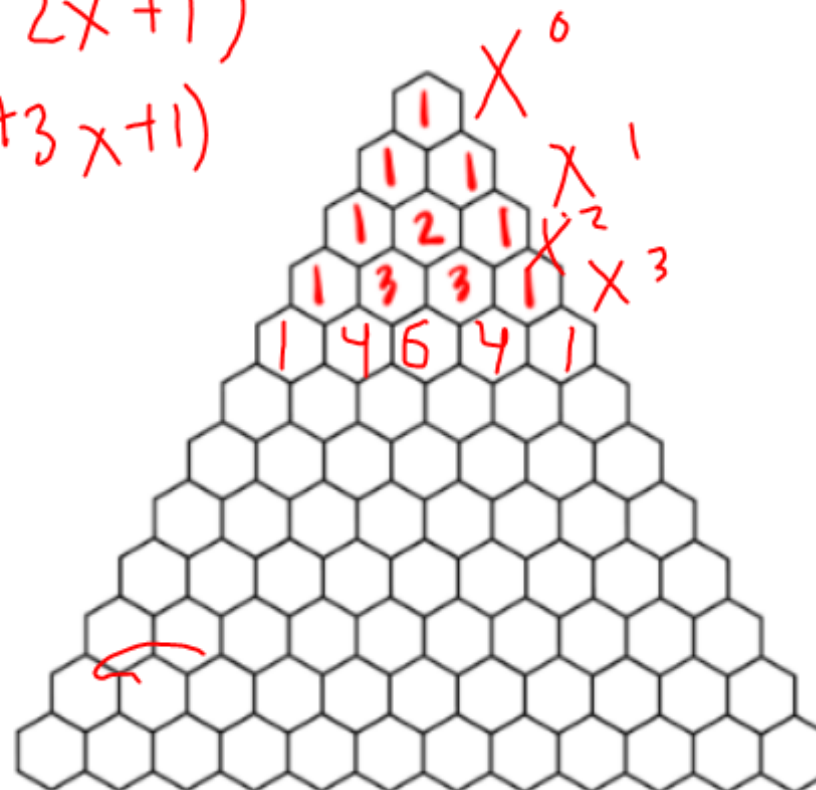
$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

# Pascal's Triangle

Pascal's Triangle is a famous mathematical pattern that has been studied for hundreds of years by mathematicians and students, and it contains many interesting patterns. The concept behind the triangle is simple: each cell in the triangle is the sum of the two numbers above it. Fill in the next three rows of the triangle.

$$(x^2 + 2x + 1)$$

$$(x^3 + 3x^2 + 3x + 1)$$

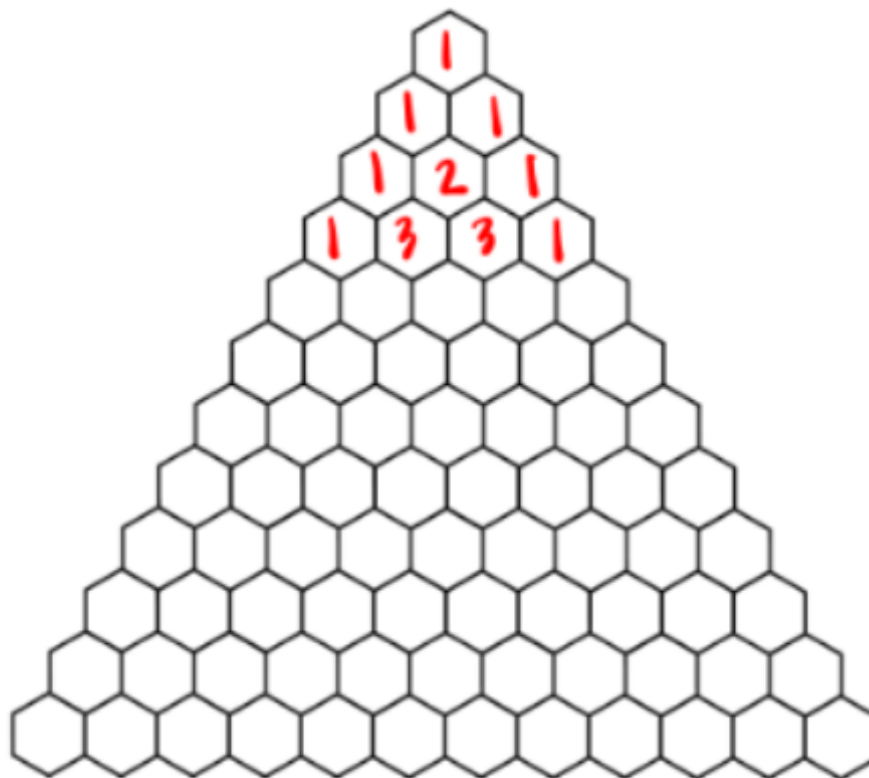


# Compare

$$(x+1)^2 =$$

$$(x+1)^3 =$$

$$(x+1)^4 =$$





## Let's go back

How can we use Pascal's Triangle to answer:

$$(x+1)^7$$

$$(x+1)^n$$

# In the spirit of generalizing..

$$(a+b)^2$$

$$(a+b)(a+b)$$

$$a^2 + 2ab + b^2$$

$$(a+b)^3$$

# Connecting the Dots

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1a + 1b

1a<sup>2</sup> + 2ab + 1b<sup>2</sup>

1a<sup>3</sup> + 3a<sup>2</sup>b + 3ab<sup>2</sup> + 1b<sup>3</sup>

# Understanding the Binomial Theorem

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

*Handwritten notes:*

- $n=3$  (written above the exponent)
- Red circles around the exponent 3 in  $(a+b)^3$  and the term  $3a^2b$ .
- Red brackets under  $a^3$ ,  $3a^2b$ ,  $3ab^2$ , and  $b^3$  with labels 0, 1, 2, 3 respectively.
- Diagram showing the expansion of  $a^{n-k} b^k$  for  $n=3$ :
  - $a^{3-0} b^0 = a^3$
  - $a^{3-1} b^1 = a^2 b$
  - $a^{3-2} b^2 = a b^2$
  - $a^{3-3} b^3 = b^3$
- Text: "each term =  $a^{3-k} b^k$ "
- Equation:  $a^1 b^2 = ab^2$

# Understanding the Binomial Theorem

*- exp*

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

*Factorial*

$n=3$     $k=1$

$4! = 4 \cdot 3 \cdot 2 \cdot 1 =$

$\binom{3}{1} = \frac{3!}{1!(3-1)!} = \frac{3 \cdot 2 \cdot 1}{1 \cdot 2 \cdot 1} = \frac{6}{2}$

$\rightarrow 3$



## Let's Apply This

$$(a+b)^3 = \underbrace{a^3}_{k^0} + \underbrace{3a^2b}_{k^1} + \underbrace{3ab^2}_{k^2} + \underbrace{b^3}_{k^3}$$

# FBO (Facebook Official)

add 'em' up!

$$\sum_{k=0}^{n-1} \binom{n}{k} a^{n-k} b^k$$

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# Review

- Specific examples of binomial expansions
- We tried to generalize with Pascal's Triangle
- Determined how to find the exponents of  $a$  and  $b$  for any binomial expansion.
- Used " $n$  choose  $k$ " to determine the coefficients of each term for any binomial expansion.
- Used Summation notation to put the pieces together.



# Practice

Together, let's expand the following binomial.

$$\sum_{k=0}^n \binom{n}{k} a^{n-k} b^k \quad \underline{(x+2)^4}$$

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# Practice Examples on Notes!!