Warmup:

Expand each logarithm.

1) $\log_6 (zx^5 \cdot y^3)$

$\log_6 z + \log_6 x^5 + \log_6 y^3$

$\boxed{\log_6 z + 5\log_6 x + 3\log_6 y}$

2) $\log_2 \left(\frac{(c \cdot a)^5}{b}\right)^3 = \log_2 \left(\frac{(ca)^{15}}{b^3}\right)$

$= \log_2 \left(\frac{c^{15} \cdot a^{15}}{b^3}\right)$

$15 \cdot \log_2 c + 15 \cdot \log_2 a - 3\log_2 b$

Condense each expression to a single logarithm.

3) $6\log_8 11 + 36\log_8 10 + 6\log_8 7$

☆ $\log_8 (10^6 \cdot 11 \cdot 7)^6$ ☆

$\log_8 (11^6 \cdot 10^{36} \cdot 7^6)$

4) $4\log_7 a + 16\log_7 b + 4\log_7 c$

$\boxed{\log_7 (a \cdot b^4 \cdot c)^4}$

☆ $\log_7 (a^4 \cdot b^{16} \cdot c^4)$ ☆

Essential Question: How can we use properties that we have learned to solve exponential and logarithmic equations?

Recall: One-to-One Property for Exponents

If $b^x = b^y$, then $x = y$.

$5^x = 25$

$5^x = 5^2$

$x = 2$

How can that apply to logarithms?

If $\log_b(x) = \log_b(y)$, then $x = y$.

$\log_2(8) = \log_2(x)$

$x = 8$

Other properties that will be useful for today's lesson:

Product and Quotient Properties of Logarithms:

For $m > 0$, $n > 0$, $b > 0$, and $b \neq 1$

☆ Product Property: $\log_b(mn) = \log_b m + \log_b n$

☆ Quotient Property: $\log_b(m/n) = \log_b m - \log_b n$

Power Property of Logs:

For $m > 0$, $b > 0$, and $b \neq 1$

☆ $\log_b m^p = p \bullet \log_b m$

I. How can you solve exponential equations using logarithms and the above property?

1. Isolate the exponential expression.
2. Take the common log of both sides.
3. Apply the power property of logs.
4. Use a calculator to approximate.
5. Check.

$a^x + 7 = 100$

$-7 \quad -7$

$a^x = 93$

base 10 log

$\log_{10} x$ or $\log x$

Examples: Solve.

$$\frac{2x}{2} = \frac{10}{2}$$

$x =$

1. $5^x = 62$

$$\log(5^x) = \log(62)$$

$$\frac{x \cdot \cancel{\log(5)}}{\cancel{\log(5)}} = \frac{\log(62)}{\log(5)}$$

$$x = \frac{\log(62)}{\log(5)}$$

$$x = 2.564$$

$$5^{2.564} = 62$$

2. $9^x = 12$

$$\log(9^x) = \log(12)$$

$$\frac{x \cdot \log(9)}{\log(9)} = \frac{\log(12)}{\log(9)}$$

$$x = \frac{\log 12}{\log 9}$$

$$x = 1.1309$$

$$x \approx 1.131$$

$$9^x = 12$$

$$\ln(9^x) = \ln(12)$$

$$x \cdot \ln 9 = \ln 12$$

$$x = \frac{\ln 12}{\ln 9}$$

3. $(7^{x-4}) = 98$

$\log 7^{x-4} = \log 98$

$\frac{(x-4)\cdot \log 7}{\log 7} = \frac{\log 98}{\log 7}$

$x - 4 = \frac{\log 98}{\log 7}$

$x - 4 = 2.356$
$+4 \quad +4$

$\boxed{x = 6.356}$

4. $(6^{3y+9}) = 100$

$\log 6^{3y+9} = \log 100$

$(3y+9)\cdot \log 6 = \log 100$

$3y + 9 = \frac{\log 100}{\log 6}$

$3y + 9 = 2.57$
$-9 \quad -9$

$\frac{3y}{3} = \frac{-6.43}{3}$

$y = -2.143$

5. $90 - 5^x = 5$

$-90 \qquad -90$

$\frac{-5^x}{-1} = \frac{-85}{-1}$

$5^x = 85$

$\frac{x \cdot \cancel{\log 5}}{\cancel{\log 5}} = \frac{\log 85}{\log 5}$

$x = \frac{\log 85}{\log 5} = 2.76$

6. $6 + 4^x = -20$

1) $9(12^r)-2=34$

$+2 \quad +2$

$\frac{\cancel{9}\cdot 12^r}{\cancel{9}} = \frac{36}{9}$

$12^r = 4$

$r \cdot \log 12 = \log 4$

$r = \frac{\log 4}{\log 12}$

2) $-7\cdot 5^n+3=-60$

3) $-9 \cdot 19^{-3v} - 5 = -104$

4) $12^{-10k} + 3 = 51$

5) $10 \cdot 17^{10-5r} + 5 = 77$

$-5 \quad -5$

$\frac{\cancel{10} \cdot 17^{10-5r}}{\cancel{10}} = \frac{72}{10}$

$17^{10-5r} = 7.2$

$(10-5r) \cdot \log 17 = \log 7.2$

$10 - 5r = \frac{\log 7.2}{\log 17}$

$10 - 5r = .697$

$-10 \qquad -10$

$\frac{-5r}{-5} = \frac{-9.303}{-5}$

$r = -1.861$

6) $-7 \cdot 9^{7r-3} + 4 = -87$

$r = .595$

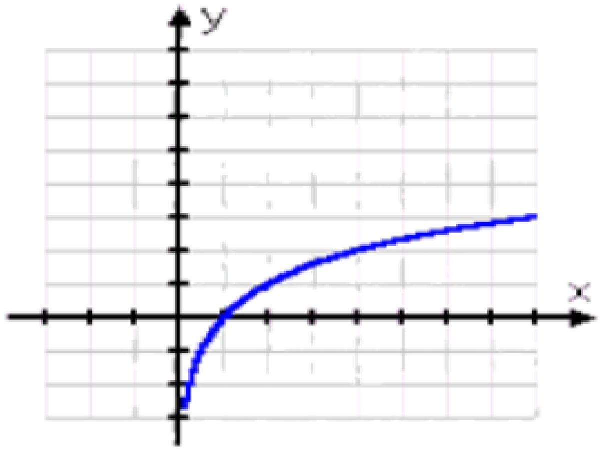
II. Recall: One-to-One Property for Exponents

If $b^x = b^y$, then x = y.

How can that apply to logarithms?

If $\log_b x = \log_b y$, then x = y.

RECALL: Domain of Logarithmic functions:



This means you can't take the log of a negative number!

Example: Solve. Check for extraneous solutions.

1. $\underbrace{\log_2}_{*} x^2 = \underbrace{\log_2}_{*} 4$

$\sqrt{x^2} = \sqrt{4}$

$x = \pm 2$ ✓

2. $(\log_5(x) - \log_5 125) = \log_5(2x - 1)$

$$\cancel{\log_5}\left(\frac{x}{125}\right) = \cancel{\log_5}(2x-1)$$

$$\cancel{125} \cdot \frac{x}{\cancel{125}} = (2x-1) \cdot 125$$

$$x = 25\cancel{0}x - 125$$

$$\begin{array}{rcl} -250x & & -\cancel{250x} \\ \hline -\cancel{249}x & = & -125 \\ \overline{-\cancel{249}} & & \overline{-249} \end{array}$$

$x = .502$

3. $\cancel{\log_2}(2x^2 - 5x - 4) = \cancel{\log_2}(8)$

$2x^2 - 5x - 4 = 8$

$\quad\quad\quad\quad \cancel{-8} \quad -8$

$2x^2 - 5x - 12 = 0$

"poly solve"

$x = -3/2$ or -1.5

$x = 4$

you can't take the log of a negative number!

4. $\cancel{\log_3}(x^2 + 7x - 5) = \cancel{\log_3}(6x + 1)$

$x^2 + 7x - 5 = 6x + 1$

$-6x \quad -1 \qquad -6x \quad -1$

$x^2 + 1x - 6 = 0$ poly solve

$x = 2$ ✓

~~$x = -3$~~

$x=2$ $\quad \log_3(2^2 + 7(2) - 5) = \log_3(6 \cdot 2 + 1)$

$\log_3(13) = \log_3(13)$

$x=-3$ $\quad \log_3((-3)^2 + 7(-3) - 5) = \log_3(6 \cdot -3 + 1)$

$\log_3(-17) = \log_3(-17)$ No negatives allowed.

$-3\cdot 7 - 3 = 18$

1) $\log_7(-4n-10) = \log_7(-3n-3)$

$\log_7(18)$

$-4n - 10 = -3n - 3$

$+10 \qquad +10$

$-4n = -3n + 7$

$+3n \qquad +3n$

$\frac{-n}{-1} = \frac{7}{-1}$

$n = -7$ ✓

$\log_2(-4)$

-4

$-8+4$

2) $\cancel{\log_2}(p) = \cancel{\log_2}(2p+4)$

$p = 2p + 4$

$-2p \qquad -2p$

$\frac{-p}{-1} = \frac{4}{-1}$

$\cancel{p = -4}$

No Solutions!

3) $\log_9(7x-2)=\log_9(x^2+4)$

4) $\log_2(7+2x^2)=\log_2(3x^2-6x)$

5) $\log_4 5x - \log_4 5 = \log_4 40$

$\log_7 3.9$

$(-5.1+9)$

6) $\underbrace{\log_7 (x+9) + \log_7 10} = \log_7 39$

$\log_7 (10 \cdot (x+9)) = \log_7 39$

$\cancel{\log_7} (10x+90) = \cancel{\log_7} 39$

$10x + 90 = 39$

$\quad -90 \quad -90$

$\frac{10x}{10} = \frac{-51}{10}$

✓ $x = -5.1$

7) $\log_6 2 + \log_6 (x^2 + 4) = \log_6 80$

8) $\log_4 5x^2 - \log_4 5 = \log_4 49$

Solving Log Equations

55. $\log_2 7x = \log_2(x^2 + 12)$

56. $\log_5(3x^2 - 1) = \log_5 2x$

57. $\log_b(x^2 - 15) = \log_b(6x + 1)$

58. $\log_{10}(5x - 3) - \log_{10}(x^2 + 1) = 0$

59. $2 \log_a x + \log_a 2 = \log_a(5x + 3)$

60. $\log_b(x^2 - 2) + 2 \log_b 6 = \log_b 6x$

61. $2 \log_3 x + \log_3 5 = \log_3(14x + 3)$

62. $\log_5 2 + 2 \log_5 t = \log_5(3 - t)$

55. $x = 3$ $x = 4$

56. $x = \cancel{-1/3}$ $x = 1$

57. No Solutions $x = \cancel{-8}$ $x = \cancel{2}$

58. $x = 4$ $x = 1$

59. $x = \cancel{-1/2}$ $x = 3$

60. $x = \cancel{-4/3}$ $x = 3/2$

61. $x = \cancel{-1/5}$ $x = 3$

62. $x = \cancel{-3/2}$ $x = 1$

Solve each equation. Show your work.

10. $4^x = 17$ **11.** $2^x = 49$ **12.** $7^x = 908$

13. $8^x = 240$ **14.** $3.5^x = 28$ **15.** $7.6^x = 64$

16. $25^x = 0.04$ **17.** $3^x = 0.26$ **18.** $2^{-x} = 0.045$

19. $7^{-x} = 0.022$ **20.** $3^x = 0.45$ **21.** $5^x = 1.29$

22. $2^{x+1} = 30$ **23.** $3^{x-6} = 81$ **24.** $11 - 6^x = 3$

25. $67 - 2^x = 39$ **26.** $8 + 3^x = 10$ **27.** $1 + 5^x = 360$

11. ~~[illegible]~~ x = 5.61

13. x = 2.64

15. x = 2.05

17. x = -1.23

19. x = 1.96

21. x = .16

23. x = 10

25. x = 4.81

27. x = 3.66