Warmup:

Solve each equation.

$7 + \log_2(-3n - 5) = 10$

$-7 \qquad -7$

$\log_2(-3n-5) = 3$

$2^3 = -3n-5$

$8 = -3n-5$

$13 = -3n$

$\overline{-3} \quad \overline{-3}$

$n = -\frac{13}{3}$ or $-4.\overline{3}$

$3\log_5(2x-8) = 3$

$\overline{3} \qquad \overline{3}$

$\log_5(2x-8) = 1$

$5^1 = 2x-8$

$5 = 2x-8$

$13 = 2x$

$x = 6.5$

Expand each logarithm.

1. $\log_6 z^5\sqrt{x}$

$\log_6 z^5 + \log_6 x^{1/2}$

$5 \cdot \log_6 z + \frac{1}{2} \cdot \log_6 x$

2. $\log_8\left(\frac{a^2}{b}\right)^6$

$\log_8 \left(\frac{a^{12}}{b^6}\right)$

$\log_8 a^{12} - \log_8 b^6$

$12 \cdot \log_8 a - 6\log_8 b$

Expand each logarithm.

1. $\log_5 x^6\sqrt{y}$

$6\log_5 x + \frac{1}{2}\log_5 y$

2. $\log_8\left(\frac{a^3}{b}\right)^4$

$\log_8 \left(\frac{a^{12}}{b^4}\right)$

$12\log_8 a - 4\log_8 b$

Condense each logarithm.

3. $\log_4 7 + \frac{\log_4 11}{2} + \frac{\log_4 3}{2}$

$\log_4 \boxed{7} + \frac{1}{2}\log_4 11 + \frac{1}{2}\log_4 3$

$\log_4 \left(7 \cdot 11^{1/2} \cdot 3^{1/2}\right)$

4. $25\log_5 a - 5\log_5 b$

$\log_5 \left(\frac{a^{25}}{b^5}\right)$

$\log_5 \left(\frac{a^5}{b}\right)^5$

$\log_2 16 = x$

$2^x = 16$

Evaluate each expression.

$(\log_2 16) + (\log_6 36) = \underline{\underline{6}}$

$4 + 2$

$\log_3 243 + \log_5 125$

$5 + 3$

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CHANGE OF BASE

How could we approximate $\log_5 3 = x$?

We can convert this to an exponential equation:

$\log 5^x = \log 3$

How can we solve this for x?

$$\log 5^x = \log 3$$

$$x \cdot \log 5 = \log 3$$

$$x = \frac{\log 3}{\log 5} = .683$$

Let's try to derive a formula by doing the same thing with $\log_b m = x$.

$$\log_b m = x$$

$$b^x = m$$

$$\log(b^x) = \log m$$

$$\frac{x \cdot \log(b)}{\log b} = \frac{\log(m)}{\log(b)}$$

$$\boxed{x = \frac{\log m}{\log b}}$$

$$\log_b m = x$$

The change of base formula is...

$$log_b m = \frac{log m}{log b}$$

Now we can use our calculator to approximate!

Example: Approximate.

1. $\log_7 56 = x$

$$\frac{\log 56}{\log 7} = x \approx 2.068$$

2.069

2. $\log_5 0.4$

$$x = \frac{\log .4}{\log 5}$$

$x \approx -.569$

3. $\log_9 378$

$$\frac{\log 378}{\log 9} = 2.701$$

4. $\log_2 100$

$$\frac{\log 100}{\log 2} = 6.644$$

5. $\left(\log_{1/2} 9\right) + 7$

$\left(\frac{\log 9}{\log 1/2}\right) + 7 = 3.830$

6. $3 - \left(\log_6 1254\right)$

$\underbrace{3 - 3.981}$

$-.982$

7. $\log_3 8 + \log_3 7$

1.092

1.893

☆ 3.664

8. $\log_7 100 - \log_2 5$

0.045

The Natural Log

$\log_2 4 = x$

$\log 4 = x$

$\ln 4 = x$

$e^x = 4$

The Natural Log is a base e logarithm.

We can write it as follows: $\log_e(x) = y$ or $\ln(x) = y$

This means:

$$e^y = x$$

or, what power do we raise e to so that we get x "e" is a number. Find the "e" button on your calculators now.

$e^{\square}$ $e^1 = 2.7182....$

$e \approx 2.718$

Base e logarithms, or Natural Logs, are still logarithms, so all of the properties we use for logs apply for ln's.

What are the properties of logs we know?

RECALL

Common Logs are in base

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NEW INFO

Natural Logs are in base

e

$e \approx$ 2.718

$(\log_e e) =$ 1

$\frac{\log e}{\log e}$

$\ln e =$ 1

Example: Evaluate.

1. $e^8 \approx 2980.960$

2. $2e^4 \approx$ ~~109.2~~

109.1963

$\boxed{109.196}$

3. $\ln 35 \approx 3.556$

4. $\ln(-1.4)$

No Solutions

Example. Solve using natural logs.

3. $33^x = 74$

$\ln 33^x = \ln 74$

$x \cdot \ln 33 = \ln 74$

$x = \dfrac{\ln 74}{\ln 33} \approx \cancel{1.2310}$

1.231

4. $4^{\frac{2}{3}x} = 0.5$

$\ln 4^{\frac{2}{3}x} = \ln .5$

$\frac{2}{3}x \cdot \ln 4 = \ln .5$

$\frac{2}{3}x = \left(\dfrac{\ln .5}{\ln 4}\right)$

$\dfrac{\frac{2}{3}x}{\frac{2}{3}} = \dfrac{-.5}{\frac{2}{3}}$

$\boxed{x = -.75}$

5. $15^{-x} = 24$

$\ln 15^{-x} = \ln 24$

$-x \ln(15) = \ln(24)$

$-x = \dfrac{\ln 24}{\ln 15}$

$\dfrac{-x}{-1} = \dfrac{1.174}{-1}$

$\boxed{x = -1.174}$

6. $0.25^{2x} = 41$

$\ln .25^{2x} = \ln 41$

$2x \ln .25 = \ln 41$

$2x = \dfrac{\ln 41}{\ln .25}$

$\dfrac{2x}{2} = \dfrac{-2.679}{2}$

$\boxed{x = -1.339}$

Assignment

Change of base, Natural log, and e WS