Warmup:

Solve each equation by square roots

$6r^2 - 9 = 87$

$+9 \quad +9$

$\frac{6r^2}{6} = \frac{96}{6}$

$\sqrt{r^2} = \sqrt{16}$

$r = \pm 4$

$2(a-6)^2 - 45 = 53$

$+45 \quad +45$

$\frac{2\cdot(a-6)^2}{2} = \frac{98}{2}$

$\sqrt{(a-6)^2} = \sqrt{49}$

$a - 6 = \pm 7$

$+6 \quad +6$

$a = 6 \pm 7$

$a = 13 \text{ or } -1$

1) $m^2 = 81$

$m = \pm\sqrt{81}$

$m = -9$ or 9

2) $p^2 = 82$

$p = \pm\sqrt{82} = \pm\sqrt{4 \cdot 41} = \pm 2\sqrt{41}$

$p = -2\sqrt{41}$ or $2\sqrt{41}$

3) $b^2 + 4 = 76$

$b^2 = 72$

$b = \pm\sqrt{72} = \pm\sqrt{36 \cdot 2} = \pm 6\sqrt{2}$

$b = -6\sqrt{2}$ or $6\sqrt{2}$

4) $x^2 + 3 = 27$

$x^2 = 24$

$x = \pm\sqrt{24} = \pm\sqrt{4 \cdot 6} = \pm 2\sqrt{6}$

$x = -2\sqrt{6}$ or $2\sqrt{6}$

5) $m^2 + 8 = 33$

$m^2 = 25$

$m = \pm\sqrt{25} = \pm 5$

$m = -5$ or 5

6) $-9x^2 = 504$

$x^2 = -56$

$x = \pm\sqrt{-56}$ = No real roots

7) $8v^2 + 4 = 444$

$8v^2 = 440$

$v^2 = 55$

$v = \pm\sqrt{55}$

$v = -\sqrt{55}$ or $\sqrt{55}$

8) $5m^2 + 5 = 90$

$5m^2 = 85$

$m^2 = 17$

$m = \pm\sqrt{17}$

$m = -\sqrt{17}$ or $\sqrt{17}$

9) $8x^2 + 7 = 263$

$8x^2 = 256$

$x^2 = 32$

$x = \pm\sqrt{32} = \pm\sqrt{16 \cdot 2} = \pm 4\sqrt{2}$

$x = -4\sqrt{2} + 4\sqrt{2}$

10) $10k^2 - 5 = -165$

$10k^2 = -160$

$k^2 = -16$

$k = \pm\sqrt{-16}$ = No real roots

11) $(x+3)^2 - 4 = 23$

$(x+3)^2 = 27$

$x+3 = \pm\sqrt{27}$

$x = \pm\sqrt{27} - 3$

$x = \pm\sqrt{9\cdot3} - 3 = \pm 3\sqrt{3} - 3$

$-3 \pm 3\sqrt{3}$

$x = -3\sqrt{3} - 3$ or $x = 3\sqrt{3} - 3$

12) $2(x-6)^2 = 50$

$(x-6)^2 = 25$

$x-6 = \pm\sqrt{25}$

$x-6 = \pm 5$

$x-6 = -5$ or $x-6 = 5$

$x = 1$ or $x = 11$

13) $-\frac{1}{2}(x+2)^2 + 8 = 12$

$-\frac{1}{2}(x+2)^2 = 4$

$(x+2)^2 = -8$

$x+2 = \pm\sqrt{-8}$

No real roots

14) $(x-5)^2 - 15 = 30$

$(x-5)^2 = 45$

$x-5 = \pm\sqrt{45}$

$x = 5 \pm \sqrt{45} = 5 \pm \sqrt{9\cdot5}$

$x = 5 \pm 3\sqrt{5}$

$x = 5-3\sqrt{5}$ or $x = 5+3\sqrt{5}$

15) $3(x+6)^2 - 7 = -21$

16) $\frac{1}{4}(x+6)^2 + 4 = 10$

15) $3(x+6)^2 - 7 = -21$

$3(x+6)^2 = -14$

$(x+6)^2 = -14/3$

$x+6 = \pm\sqrt{-14/3}$ No real roots

16) $\frac{1}{4}(x+6)^2 + 4 = 10$

$\frac{1}{4}(x+6)^2 = 6$

$(x+6)^2 = 24$

$x+6 = \pm\sqrt{24}$

$x = \pm\sqrt{24} - 6$

$x = \pm\sqrt{4 \cdot 6} - 6$

$x = \pm 2\sqrt{6} - 6$

$x = -2\sqrt{6} - 6$ or $x = 2\sqrt{6} + 6$

17) $-(x-10)^2 - 4 = 12$

18) $2(x-7)^2 + 9 = 107$

$\sqrt{\frac{25}{4}} =$

17) $-(x-10)^2 - 4 = 12$

$-(x-10)^2 = 16$

$(x-10)^2 = -16$

$(x-10) = \pm\sqrt{-16}$

No real roots

18) $2(x-7)^2 + 9 = 107$

$2(x-7)^2 = 98$

$(x-7)^2 = 49$

$x-7 = \pm 7$

$x = 7 \pm 7$

$x = 7+7 = 14$

or

$x = 7-7 = 0$

19) $(x+6)^2 - 4 = 40$

$(x+6)^2 = 44$

$x+6 = \pm\sqrt{44}$

$x = \pm\sqrt{44} - 6$

$x = \pm\sqrt{4 \cdot 11} - 6$

$x = \pm 2\sqrt{11} - 6$

$x = -2\sqrt{11} - 6$

$x = 2\sqrt{11} - 6$

20) $-7(x-8)^2 = 112$

$(x-8)^2 = -16$

$x-8 = \pm\sqrt{-16}$

No real roots

E.Q.:

How do we solve quadratic equations using the completing the square method?

Factor each of the following:

Squares

$x^2 + 2x + 1 = (x+1)(x+1) = (x+1)^2$

$x^2 - 4x + 4 = (x-2)(x-2) = (x-2)^2$

$x^2 + 6x + 9 = (x+3)(x+3) = (x+3)^2$

$x^2 - 10x + 25 = (x-5)(x-5) = (x-5)^2$

Remember our perfect square pattern:

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

Let's say we want to solve the following equation:

$$x^2 + 10x + 25 = 10$$

$-10 \quad -10$

What do we do first?

$x^2 + 10x + 15 = 0$

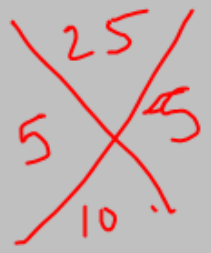
What do we next?

Factor!

$$x^2 + 10x + 15 = 0$$

It's not factorable now!!

So what do we do?



Let's try factoring the quadratic first, before we set it equal to 0!!

$$x^2 + 10x + 25 = 10$$

$$(x+5)^2 = 10$$

Solve by Sq. Roots

$$\sqrt{(x+5)^2} = \sqrt{10}$$

$$x + 5 = \pm\sqrt{10}$$
$$-5 \quad -5$$

$$x = -5 \pm \sqrt{10}$$

In fact, now we don't need to set it equal to zero.

$$(x+5)^2 = 10$$

Why not?

Let's try this one:

$$x^2 + 8x + 2 = 0$$

$-2 \quad -2$

It's not factorable at all is it?

$x^2 + 8x + 16 = -2 + 16$

$(x + 4)^2 = 14$

$\frac{8}{2} = 4$

$4^2 = 16$

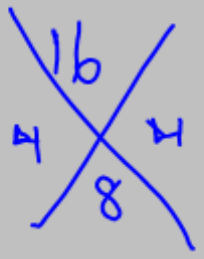
What if we move the 2 over to the other side?

$$x^2 + 8x + \underline{16} = -2 \quad +16$$

COMPLETE THE SQUARE

What value should fill in the blank so that the quadratic is a perfect square?

$$x^2 + 8x + 16 = -2 + 16$$



$$x^2 + 8x + 16 = 14$$

$(x+4)(x+4)$

$$\sqrt{(x+4)^2} = \pm\sqrt{14}$$

$x+4 = \pm\sqrt{14}$

$-4 \quad -4$

$x = -4 \pm \sqrt{14}$

Solve by completing the square:

$$x^2 - 4x + 10 = 42$$

$-10 \quad -10$

$x^2 - 4x + 4 = (x-2)(x-2)$

Factor $(x-2)^2$

$x^2 - 4x + \underline{4} = 32 + \underline{4}$

$\frac{-4}{2} = -2$

$(x-2)^2 = \sqrt{36}$

$(-2)^2 = 4$

$x - 2 = \pm 6$

$+2 \quad +2$

$x = 2 \pm 6$

$x = 8$

$x = -4$

$$x^2 + 12x - 1 = 25$$

$$x^2 + 12x + \underline{36} = 26 + \underline{36}$$

$\frac{12}{2} = 6$

$6^2 = 36$

$$\sqrt{(x + \underline{6})^2} = \sqrt{62}$$

$$x + 6 = \pm\sqrt{62}$$

$$\boxed{x = -6 \pm \sqrt{62}}$$

To use Completing the square to solve a quadratic ($ax^2 + Bx + C$) you need to ...

1. Move the **constant** (c) over to one side of the equation
2. Factor out the **leading coefficient** (if needed) of the ax^2 and bx
3. Find the number that would make the trinomial a **Trinomial Square**
4. Add that number to the other side (whatever you do to one side of an equation you need to do to the other.
    - If the leading coefficient had been factored out you need to **multiply** the number by what was factored out and add new number to the other side
5. **Factor** the trinomial square

Solve by completing the square:

$4 \cdot 1 = 4$

$$4x^2 - 8x - 32 = 0$$

$+32 \quad +32$

$4x^2 - 8x = 32$

$\frac{-2}{2} = -1$

$4(x^2 - 2x + 1) = 32 + 4$ $\quad (-1)^2 = 1$

$\frac{4(x-1)^2}{4} = \frac{36}{4}$

$\sqrt{(x-1)^2} = \sqrt{9}$

$x - 1 = \pm 3$

$+1 \quad +1$

$x = 1 \pm 3$

$x = 4$

$x = -2$

You try:

$$2x^2 - 12x - 10 = 44$$

+10 +10

$2x^2 - 12x = 54$ - - -

$2(x^2 - 6x + 9) = 54 + 18$

$\frac{-6}{2} = -3$

$(-3)^2 = 9$

$\frac{2(x-3)^2}{2} = \frac{72}{2}$

$\sqrt{(x-3)^2} = \sqrt{36}$

$x - 3 = \pm 6$

$x = 3 \pm 6$

$x = 9$
$x = -3$

HW #6:
Solving Quadratic Equations Using Completing the Square