Warmup: Solve by completing the square

$$3x^2 - 12x + 6 = 0$$
$$-6 \quad -6$$

$$3x^2 - 12x = -6$$

$$3(x^2 - 4x + \underline{4}) = -6 + \underline{12}$$

$$\frac{3(x-2)^2}{3} = \frac{6}{3}$$

$$\sqrt{(x-2)^2} = \sqrt{2}$$

$$x - 2 = \pm\sqrt{2}$$

$$\boxed{x = 2 \pm \sqrt{2}}$$

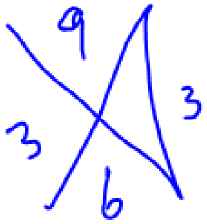
4, -2, -2, -4

$$\frac{-4}{2} = -2$$

$$(-2)^2 = 4$$

$$x^2 + 6x - 7 = 0$$

$$+7 \qquad +7$$



$$x^2 + 6x + \underline{\ 9\ } = 7 + \underline{\ 9\ }$$

$$\sqrt{(x+3)^2} = \sqrt{16}$$

$$\frac{6}{2} = 3$$

$$3^2 = 9$$

$$x + 3 = \pm 4$$

$$-3 \qquad -3$$

$$x = -3 \pm 4$$

$$x = 1 \text{ or } -7$$

$$x^2 + 5x + \frac{25}{4} = \frac{7 \cdot 4}{1 \cdot 4} + \frac{25}{4}$$

$$\frac{28}{4} + \frac{25}{4} = \frac{53}{4}$$

$$\left(x + \frac{5}{2}\right)^2 = \frac{53}{4}$$

$$\left(\frac{5}{2}\right)^2 =$$

$$(2.5)^2$$

$$3x^2 - 12x + 6 = 0$$

1) $v^2 + 12v - 15 = 0$

$v^2 + 12v + \underline{36} = 15 + \underline{36}$

$\frac{12}{2} = 6$

$6^2 = 36$

$(v+6)^2 = 51$

$v+6 = \pm\sqrt{51}$

$v = -6 \pm \sqrt{51}$

2) $r^2 - 20r + 33 = 0$

$r^2 - 20r + \underline{100} = -33 + \underline{100}$

$\frac{-20}{2} = -10$

$(-10)^2 = 100$

$(r-10)^2 = 67$

$r-10 = \pm\sqrt{67}$

$r = 10 \pm \sqrt{67}$

3) $r^2 + 20r + 99 = 0$

$r^2 + 20r + \underline{100} = -99 + \underline{100}$

$\frac{20}{2} = 10$

$10^2 = 100$

$(r+10)^2 = 1$

$r+10 = \pm 1$

$r = -10 \pm 1$

$r = \underline{-9}$ or $\underline{-11}$

4) $v^2 - 6v + 5 = 0$

$v^2 - 6v + \underline{9} = -5 + \underline{9}$

$\frac{-6}{2} = -3$

$(-3)^2 = 9$

$(v-3)^2 = 4$

$v-3 = \pm 2$

$v = 3 \pm 2$

$v = \underline{5}$ or $\underline{1}$

5) $6r^2 - 12r - 79 = 0$

$6r^2 - 12r = 79$

$6(r^2 - 2r + \underline{1}) = 79 + \underline{6}$

$\frac{-2}{2} = -1$

$(-1)^2 = 1$

$6(r-1)^2 = 85$

$(r-1)^2 = \frac{85}{6}$

$r-1 = \pm\sqrt{\frac{85}{6}}$

$r = 1 \pm \sqrt{85/6}$

6) $5k^2 - 10k - 75 = 0$

$5k^2 - 10k = 75$

$5(k^2 - 2k + \underline{1}) = 75 + \underline{5}$

$\frac{-2}{2} = -1$

$(-1)^2 = 1$

$5(k-1)^2 = 80$

$(k-1)^2 = 16$

$k-1 = \pm 4$

$k = 1 \pm 4$

$k = 5 \text{ or } -3$

7) $10p^2 - 20p - 30 = 0$

$10p^2 - 20p = 30$

$10(p^2 - 2p + \underline{1}) = 30 + \underline{10}$

$\frac{-2}{2} = -1$

$(-1)^2 = 1$

$10(p-1)^2 = 40$

$(p-1)^2 = 4$

$p - 1 = \pm 2$

$p = 1 \pm 2$

$p = 3$ or -1

8) $3a^2 + 6a - 42 = 0$

$3a^2 + 6a = 42$

$3(a^2 + 2a + \underline{\quad}) = 42 + \underline{\quad}$

$\frac{2}{2} = 1$

$1^2 = 1$

$3(a^2 + 2a + 1) = 42 + 3$

$3(a+1)^2 = 45$

$(a+1)^2 = 15$

$a + 1 = \pm\sqrt{15}$

$a = -1 \pm \sqrt{15}$

9) $b^2 = 70 - 4b$

$b^2 + 4b + \underline{4} = 70 + \underline{4}$

$\frac{4}{2} = 2$

$2^2 = 4$

$(b+2)^2 = 74$

$b + 2 = \pm\sqrt{74}$

$b = -2 \pm \sqrt{74}$

10) $x^2 = -2x + 63$

$x^2 + 2x + \underline{1} = 63 + \underline{1}$

$\frac{2}{2} = 1$

$1^2 = 1$

$(x+1)^2 = 64$

$x + 1 = \pm 8$

$x = -1 \pm 8$

$x = 7$ or -9

11) $7v^2 - 21 = -14v$

$7v^2 + 14v = 21$

$7(v^2 + 2v + \underline{1}) = 21 + \underline{7}$

$\frac{2}{2} = 1$

$1^2 = 1$

$7(v+1)^2 = 28$

$(v+1)^2 = 4$

$v + 1 = \pm 2$

$v = -1 \pm 2$

$v = 1$ or -3

12) $5n^2 + 20n = 66$

$5(n^2 + 4n + \underline{4}) = 66 + \underline{20}$

$\frac{4}{2} = 2$

$2^2 = 4$

$5(n+2)^2 = 86$

$(n+2)^2 = \frac{86}{5}$

$n + 2 = \pm\sqrt{\frac{86}{5}}$

$n = -2 \pm \sqrt{\frac{86}{5}}$

E.Q.:

How do we solve quadratic equations using the quadratic formula?

SHORTCUT
To Completing
The Square?

The Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This formula works for all **standard form** quadratic equations.

It is directly derived by completing the square on the standard form of a quadratic equation:

$$ax^2 + bx + c = 0$$

We have studied several different ways to solve quadratic equations (factoring and taking square roots). These methods may not work in certain instances. The **Quadratic Formula** is a method of solving quadratics that **works for every quadratic equation.**

- -In order to set up quadratic formula, we need our quadratic equation written in standard form:

$$ax^2 + bx + c = 0$$

- -Once the quadratic is in standard form, we plug **the coefficients** a, b, and c into the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve using the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x^2 + 6x + 5 = 0$$

$$x = \frac{-6 \pm \sqrt{(6)^2 - 4(1)(5)}}{2(1)}$$

$a = 1$

$b = 6$

$c = 5$

$$x = \frac{-6 \pm \sqrt{16}}{2}$$

$$x = \frac{-6 \pm 4}{2}$$

$$x = \boxed{\frac{-6+4}{2}} = \frac{-2}{2} = -1$$

$$x = \boxed{\frac{-6-4}{2}} = \frac{-10}{2} = -5$$

EXAMPLES Set up the quadratic formula for each of the following quadratic equations.

$6x^2 - 45 = 3x$

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4(6)(^-45)}}{2(6)}$$

$6x^2 - 3x - 45 = 0$

$a = 6$

$b = -3$

$c = {}^-45$

$$x = \frac{3 \pm \sqrt{1089}}{12}$$

$$x = \frac{3 \pm 33}{12}$$

$$x = \frac{3 + 33}{12} = 3$$

$$x = \frac{3 - 33}{12} = -\frac{5}{2}$$

or

-2.5

$$4x^2 = -9 - 9x$$

$4x^2 + 9x + 9 = 0$

$a = 4$

$b = 9$

$c = 9$

$$x = \frac{-9 \pm \sqrt{(9)^2 - 4(4)(9)}}{2(4)}$$

$$x = \frac{-9 \pm \sqrt{-63}}{8}$$

No real roots

$$x^2 + 6x + 6 = 0$$

$$x = \frac{-6 \pm \sqrt{(6)^2 - 4(1)(6)}}{2(1)}$$

$a = 1$

$b = 6$

$c = 6$

$$x = \frac{-6 \pm \sqrt{12}}{2}$$

$$x = \frac{-6 \pm 2\sqrt{3}}{2}$$

$$x = \frac{-6 + 2\sqrt{3}}{2} \qquad x = \frac{-6 - 2\sqrt{3}}{2}$$

$$x = \sqrt{3} - 3 \qquad x = -\sqrt{3} - 3$$

$-3 \pm \sqrt{3}$

$$5x^2 = 80$$

$$5x^2 - 80 = 0$$

$a = 5$

$b = 0$

$c = -80$

$$\frac{\cancel{0} \pm \sqrt{\cancel{0}^2 - 4(5)(-80)}}{2(5)}$$

$$\frac{\pm\sqrt{1600}}{10}$$

$$\frac{\pm 40}{10}$$

$\boxed{x = 4}$ $\boxed{x = -4}$

or ± 4

HW #7:
Solving Quadratic Equations Using the Quadratic Formula