

Warmup:



Rabbit Population Growth

In 1995, there were 85 rabbits in Central Park. The population increased by 12% each year. How many rabbits were in Central Park in 2005?

Your Tasks:

$$P_t = P_0(1+r)^t$$

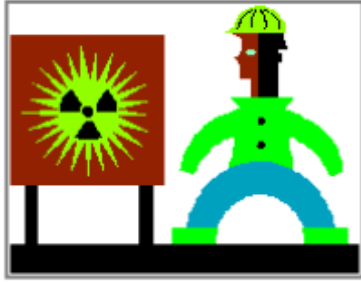
a. Complete the chart:

0	Years	1 1996	2 1997	3 1998	4 1999	5 2000	6 2001	7 2002	8 2003	9 2004	10 2005
85	Number of Rabbits (Round to the nearest rabbit.)	95	106	119	134	150	167	187	210	235	264

b. Write a function to illustrate the situation.

$$P_t = 85(1 + .12)^t$$

$$P_t = 85(1.12)^t$$



Radium Decay

In 2000, 50 grams of radium were stored. The half-life of radium is 1,620 years. How many grams of radium remains after 4860 years? Remember, half-life is the amount of time it takes for half of the amount of a substance to decay.

Your Tasks:

a. Complete the chart:

End of Half life cycle	① 1620 yrs	② 3240 yrs	③ 4860 yrs
Grams of radium remaining	25	12.5	6.25
Pattern:			

b. Write a function to illustrate the situation

$$P_t = 50(1 - 0.5)^t$$

$$P_t = P_0(1 - r)^t$$

1. The population of Brazil was about 162,661,000 in 1996 and was project to grow at a rate of 7.7% per decade. Predict the population, to the nearest hundred thousand, of Brazil for 2016 and 2020.

$$P_t = 162,661,000 (1 + 0.077)^t$$

$$2016 \Rightarrow t = 2 \quad P(2) \approx 188,675,211$$

$$2020 \Rightarrow t = 4 \quad P(4) \approx 194,357,420$$

2. A vitamin is eliminated from the bloodstream at a rate of 20% per hour. The vitamin reaches a peak level in the bloodstream of 300 milligrams. Predict the amount, to the nearest tenth of a milligram, of the vitamin remaining 2 hours and 7 hours after the peak level.

$$P_t = 300 (1 - 0.2)^t$$

$$P(2) = 192 \text{ mg}$$

$$P(7) \approx 62.9 \text{ mg}$$

3. Suppose that you put \$2500 into a retirement account that grows with an interest rate of 5.25% each year. Find the balance in the account after 5 years, 10 years, and 15 years.

$$P_t = 2500 (1 + 0.0525)^t$$

$$P(5) \approx \$3,228.87$$

$$P(10) \approx \$4,170.24$$

$$P(15) \approx \$5,386.06$$

4. The first stage of the *Saturn 5* rocket that propelled astronauts to the moon burned about 8% of its fuel every 15 seconds and carried about 600,000 gallons of fuel at liftoff. Estimate the amount of fuel remaining, to the nearest ten thousand gallons, in the first stage 1 minute, 1.5 minutes, and 2 minutes after takeoff.

$$P_t = 600,000 (1 - .08)^t$$

$$1 \text{ min} \Rightarrow t = 4 \quad P(4) \approx 430,000 \text{ gal}$$

$$1.5 \text{ min} \Rightarrow t = 6 \quad P(6) \approx 360,000 \text{ gal}$$

$$2 \text{ min} \Rightarrow t = 8 \quad P(8) \approx 310,000 \text{ gal}$$

5. Each year the local country club sponsors a tennis tournament. Play starts with 128 participants. During each round, half of the players are eliminated. How many players remain after 5 rounds?

$$P_t = 128 (1 - .5)^t$$

$$P(5) = 4 \text{ players}$$

6. You have inherited land that was purchased for \$30,000 in 1960. The value of the land increased by approximately 5% per year. What is the approximate value of the land in the year 2011?

$$P_t = 30,000 (1 + .05)^t$$

$$2011 \Rightarrow t = 51$$

$$P(51) \approx \$361,223.09$$

Quiz #10

E.Q.:



How do we apply the idea of exponential growth to compound interest?

Simple and Compound Interest

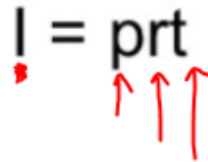
Total Real Life Stuff



Simple Interest

- When you first deposit money in a savings account, your deposit is called  **PRINCIPAL**. 
- The bank takes the money and invests it.
- In return, the bank pays you INTEREST based on the INTEREST RATE.
- Simple interest is interest paid *only* on the PRINCIPAL.

Simple Interest Formula

$$I = prt$$


- I = interest
- P = principal
- R = the interest rate per year
- T = the time *in years.*

Real-World

- Suppose you deposit \$400 in a savings account. The interest rate is 5% per year.
- Find the interest earned in 6 years. Find the total of principal plus interest.

$$I = (400)(.05)(6)$$

$$I = \$120$$

10% of \$400 = 40
5% = \$20

$$\begin{array}{r} \$20 \\ \times 6 \\ \hline \$120 \end{array}$$

$$\text{Total} = \frac{400}{\text{original}} + \frac{120}{\text{Interest}} = \$520$$

Real-World

- Suppose you deposit \$400 in a savings account. The interest rate is 5% per year.
- Find the interest earned in 6 years. Find the total of principal plus interest.
- $I = PRT \rightarrow$ Formula
- $P = 400$, $R = 0.05 = 5\%$, $T = 6$ (in years) ★
- $400 \times 0.05 = 20 =$ interest on one year
- $400 \times 0.05 \times 6 = 120 =$ interest on \$400 over 6 years
- $400 + 120 = \$520 =$ amount in account after 6 years.

To find the value of your investment directly you can use this formula:

Total Value

$$P_t = P_0 + \underbrace{(P_0 \cdot r)t}_{\text{original + interest}} \quad \star$$

↓
original + interest

$$I = prt \quad (\text{interest only})$$

Now Figure Interest In Months

- Remember that $T =$ time in Years.
- So, Find the interest earned in three months. Find the total of principal plus interest.

time (years)

$$P = 400$$

$$r = 5\%$$

$$\text{time} = \underline{\underline{3 \text{ months.}}}$$

$$I = (400)(.05)\left(\frac{3}{12}\right)$$

$$I = \$5$$

$$\text{Total Value} = 400 + 5 = 405$$

$$\underbrace{400 + (400)(.05)\left(\frac{3}{12}\right)}$$

Now Figure Interest In Months

- Remember that T = time in Years.
- So, Find the interest earned in three months. Find the total of principal plus interest.
- What fraction of a year is 3 months?

$$T = \underline{3/12} = \underline{1/4} \text{ or } \underline{0.25}$$

$$I = PRT$$

$$I = 400 \times 0.05 \times 0.25$$

$$I = \$5 = \text{interest earned after 3 months}$$

$$\$5 + \$400 = \text{total amount in account}$$

$$\$405$$

Try These: Both Find the Simple Interest

- Principal = \$250
- Interest Rate = 4%
- Time = 3 Years

$$\text{Interest} = \$ \underline{30}$$

$$\text{Total Value} = \$ 280$$



Reminder: Time is always in terms of Years. So, if you're dealing with months, you have to make your months a fraction of a year.

- Principal = \$250
- Interest Rate = 3.5%
- Time = 6 Months

$$\text{Interest} = \$ 4.38$$

$$\text{Total Value} = \$ \underline{254.38}$$

Compound Interest

- ***Compound Interest*** is when the bank pays interest on the Principal **AND** the Interest already earned.
- ***The Balance*** is the Principal PLUS the Interest.
- The Balance becomes the Principal on which the bank figures the next interest payment when doing Compound Interest.

Compound Interest Example

- You deposit \$400 in an account that earns 5% interest compounded annually (once per year). What is the balance in your account after 4 years? In your last calculation, round to the nearest cent.

$$400 @ 5\% = \$20$$

Fill In This Chart

$$400 \times .05 = \cancel{20} 20$$

$$420 \times .05 = 21$$

$$441 \times .05 = 22.05$$

$$463.05 \times .05 = 23.15$$

Principle @ Beginning of Year	Interest (I = PRT) <i>simple</i>	Balance at End of Each Year <i>compound</i>
Year 1: \$400.00	\$ 420.00	\$ 420.00
Year 2:	\$ 440.00	\$ 441.00
Year 3:	\$ 460.00	\$ 463.05
Year 4:	\$ 480.00	\$486.20

Compound Interest Formula

- You can find a balance using compound interest in one step with the compound interest formula.
- An INTEREST PERIOD is the length of time over which interest is calculated.
- The Interest Period can be a year or less than a year.

Growth

$$P_t = P_0(1+r)^t$$

$$P_t = P_0 \left(1 + \frac{r}{n}\right)^{nt}$$

$P_t =$ value at time t
 $P_0 =$ initial investment

$r =$ interest rate
 $n =$ number of interest periods

$t =$ time

Example

- Find the balance on a deposit of \$1,000, earning 6% interest compounded semiannually for 5 years.

$$P_0 = 1000$$

$$r = 6\% \text{ or } .06$$

$$n = \text{semi-annually} = 2 \text{ times}$$

$$t = 5$$

$$P_t = 1000 \left(1 + \frac{.06}{2} \right)^{2 \cdot 5}$$

$$P_t = \underline{\underline{\$1343.91}}$$

Simple 6% 5yrs

$$I = 1000(.06)(5)$$

$$I = 300$$

$$\text{Total} = \$1300$$

Try These: Both

- Find the balance for each account. Amount Deposited: \$900, Annual Interest Rate: 2%, Time: 3 Years.



- Compounding Annually
- Compounding Semiannually

$$P_t = 900 \left(1 + \frac{.02}{1} \right)^{1 \cdot 3} = 955.09$$

$$P_t = 900 \left(1 + \frac{.02}{2} \right)^{2 \cdot 3} = 955.37$$

HW #6

Simple vs Compound Interest