

Simplifying Rational Expressions

Examples of rational expressions

$$\frac{4}{3x} \quad \frac{x-8}{x+3} \quad \frac{4y-7}{y^2+5y-9}$$

Steps to Simplifying a Rational Expression:

SIMPLIFY: $\frac{7x-7}{x^2-1} = \frac{7(x-1)}{(x+1)(x-1)} = \frac{7}{x+1}$

(Handwritten notes: $(x+1)(x-1) = x^2-1$)

(Handwritten note: $\frac{7}{x+1}$ is circled in red)

Step 1: Factor the numerator and the denominator completely looking for common factors.

Step 2: Divide the numerator and denominator by the common factor.

Step 3: Multiply across to get your answer.

$$\frac{12}{3} = \textcircled{4}$$

~~12~~

$$\frac{-12}{3} = \textcircled{-4}$$

$$\frac{12}{0} = \text{undefined}$$

$$\frac{0}{12} = 0 \checkmark$$

$$\frac{12}{1} = 12$$

$$\frac{12}{12} = 1$$

$$\text{So, } f(x) = \frac{7}{x+1} = \frac{7}{-1+1} \neq \frac{7}{0} \quad \frac{7}{1+1} = \frac{7}{2} = 3.5$$

But x can not be +1 or -1.
Why?

Be sure you use the original denominator when finding these values!

Simplify:

$$\frac{2a-6}{10} = \frac{2(a-3)}{5 \cancel{2}} = \frac{a-3}{5}$$

$$\frac{a-3}{5}$$

Simplify:

$$\frac{(a^2 - 10a + 21)}{a^2 - 11a + 28} = \frac{(a-3)(\cancel{a-7})}{(\cancel{a-7})(a-4)} = \frac{a-3}{a-4}$$

$$\begin{array}{ccc} & 21 & \\ -3 & \times & -7 \\ & -10 & \end{array}$$

$$\begin{array}{ccc} & 28 & \\ -7 & \times & -4 \\ & -11 & \end{array}$$

$$a \neq 7 \text{ or } 4$$

Simplify:

$$\frac{3a^3 - a^4}{2a^3 - 6a^2} = \frac{\cancel{a}^3 (3 - a)}{2\cancel{a}^2 (a - 3)}$$

$$= \frac{a(3-a)}{2(a-3)}$$

$$\frac{a^3}{2a^2}$$

The expression is undefined when:

$$a \neq 3 \text{ or } 0$$

$$2a^2 \cdot (a - 3) = 0$$

$$\frac{2a^2}{2} = 0$$

$$\sqrt{a^2} = \sqrt{0}$$

$$a = 0$$

$$a - 3 = 0$$

$$a = 3$$

#'s 11 through 20
from yesterday's
worksheet

MULTIPLYING AND DIVIDING RATIONAL EXPRESSIONS NOTES

Recall how to **MULTIPLY** Fractions ...

1 – multiply the numerators

2 – multiply the denominators

$$\frac{5}{6} \cdot \frac{2}{20} = \frac{10}{120} = \frac{1}{12}$$

$$\frac{\cancel{5}}{6} \cdot \frac{\cancel{2}}{\cancel{20} 10 \cdot 2} = \frac{1}{12}$$

Now try it with rational expressions:

$$\frac{\cancel{4}a^2}{5ab^3} \cdot \frac{\cancel{3}bc}{\cancel{12}a^3} = \frac{1c}{5a^2b^2}$$

b^2 4 a

$$\frac{c}{5a^2b^2}$$

Sometimes we will need to factor first:

Step 1: **Factor** the numerator and the denominator

Step 2: **Divide** the numerator and denominator by the **common factors**

Step 3: Multiply the numerator and the denominator across
list what values **cannot** be in the denominator

$$\frac{x^2(x+3)}{(x^2+3x+2)} \cdot \frac{(x-9)(x-1)}{(x^2-10x+9)}$$

$$\frac{x^2(x+3)}{(x+6)(x-1)} \cdot \frac{(x-9)(x-1)}{(x-9)(x+3)}$$

$$= \frac{x^2}{x+6}$$

$x \neq -6, 1, 9, -3$

Handwritten notes and diagrams:
 - A blue star is above the final simplified fraction.
 - Three red diagrams show the cancellation of common factors:
 1. $\begin{matrix} 9 & & -1 \\ -9 & \times & -10 \end{matrix}$
 2. $\begin{matrix} -27 & & 3 \\ -9 & \times & -6 \end{matrix}$
 3. $\begin{matrix} -6 & & -1 \\ 6 & \times & 5 \end{matrix}$

$$\frac{\cancel{2} \cdot 4}{\cancel{m+3}} \cdot \frac{\cancel{2} \cdot \cancel{(m+3)} \cdot 2m+6}{\cancel{2}} = \textcircled{4} \quad \underline{\underline{m \neq -3}}$$

$$m=0$$

$$\frac{4}{(m+3)} \cdot \frac{2m+6}{2} = \frac{4}{3} \cdot \frac{6}{2} = \frac{24}{6} = \textcircled{4}$$

$$\frac{\cancel{p+8} \quad p+1}{\cancel{p^2+9p+8}} \cdot \frac{5}{p-2} = \frac{5(p+1)}{p-2} \quad p \neq 2, -8$$

$$\begin{array}{c} 8 \\ \diagdown \quad \diagup \\ 8 \quad 1 \\ \diagup \quad \diagdown \\ 9 \end{array}$$

$$\frac{n-6}{\cancel{2n^2}} \cdot \frac{\cancel{2n^2}(n-1)}{\cancel{2n^3-2n^2}} = n-6 \quad n \neq 1, 0$$

Recall Dividing with Fractions:

Multiply by the Reciprocal.

Keep, Change, Flip

$$\frac{4}{5} \div \frac{16}{25}$$

$$\frac{\cancel{4}}{\cancel{5}} \cdot \frac{\cancel{25}^5}{\cancel{16}_4} = \left(\frac{5}{4} \right)$$

We can also divide with rational expressions the same way:

K, C, F or **Outers Over Innners**

Factor the numerator and the denominator

Divide the numerator and denominator by the **common factors**

Multiply the numerator and the denominator across

List what values **cannot** be in the denominator

$$\frac{y+2}{y^2+10y-24} \div \frac{y^2+2y}{y^2+2y-8}$$

$$\begin{array}{r} 27 \\ 12 \times -2 \\ \hline 10 \end{array}$$

$$= \frac{\cancel{(y+2)}}{\cancel{y^2+10y-24}} \cdot \frac{(y+4)(y-2)}{y(y+2)}$$

$$\frac{(y+4)(\cancel{y-2})}{\cancel{(y^2+2y-8)}} \cdot \frac{\cancel{(y^2+2y)}}{y(\cancel{y+2})}$$

$$\left. y \neq -12, 2, -2, 0 \quad \frac{y+4}{y(y+12)} \right\}$$

$$\begin{array}{r} -8 \\ +4 \times -2 \\ \hline 2 \end{array}$$

$$\frac{x-3}{x^2+4x-12} \div \frac{2x^2-6x}{x-2}$$

$$\frac{(x-3)}{(x+6)(x-2)} \cdot \frac{x-2}{2x(x-3)} = \frac{1}{2x(x+6)}$$

$$x \neq -6, 2, 0, 3$$

HW #1: Rational Expressions