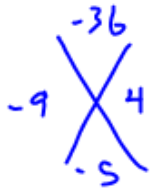
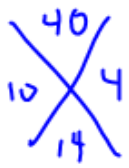
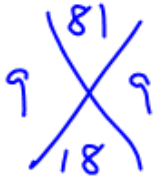
Warmup:







Simplify each and state the excluded values.

1) $\dfrac{p^2-5p-36}{p^2+14p+40}$ $= \dfrac{(p-9)(p+4)}{(p+10)(p+4)}$

$p \neq -10, -4$

$= \dfrac{p-9}{p+10}$

2) $\dfrac{v^2+18v+81}{8v+72}$

$= \dfrac{(v+9)(v+9)}{8(v+9)}$

$= \dfrac{v+9}{8}$ $v \neq -9$

Quiz

Solving Rational Equations

A rational equation is an equation containing one or more rational expressions.

proportions

$\frac{1}{x} = \frac{1}{4-x}$ $x + \frac{6}{x} = -5$ $\frac{3}{4} + \frac{5}{x} = \frac{x}{x+2}$

$x \neq 4$ or 0 $x \neq 0$ $x \neq 0$ or -2

Since the denominator of a fraction can NEVER be equal to zero, there are excluded values for the variable in a rational expression or equation. What are the excluded values for the above examples?

Solving Rational Equations (2 types of equations are possible)

1) $\frac{1}{x} = \frac{1}{4-x}$ $x \neq 0$ or 4

$1 \cdot x = 1 \cdot (4-x)$

$x = 4 - x$

$+x \quad +x$

$\frac{2x}{2} = \frac{4}{2}$

$x = 2$

2) $x + \frac{6}{x} = -5$

$x \cdot \left(\frac{x}{1} + \frac{6}{x} \right) = -5 \cdot x$

$\frac{x^2}{x} + \frac{6}{x} = -5$

$\frac{x^2+6}{x} = \frac{-5}{1}$

$1(x^2+6) = -5 \cdot x$

$x^2 + 6 = -5x$

$+5x \quad +5x$

$x^2 + 5x + 6 = 0$

$(x+2)(x+3) = 0$

$x = -2 \quad x = -3 \quad x \neq 0$

$\frac{1}{2} = \frac{x}{6}$

$x = 3$

$\frac{6}{2} = \frac{2x}{2}$

$6 = 2x$

$3 = x$

To solve any rational equation we want to clear out the denominators.

1) $\frac{1}{x} = \frac{1}{4-x}$

To solve a proportion, we want to simply cross multiply and solve the resulting equation.

2) $x + \frac{6}{x} = -5$

To solve a rational equation that is not a proportion, we want to multiply every term in the equation by the Lowest Common Multiple (LCM) of every denominator.

This will clear out the denominators, allowing us to solve the resulting equation.

$\frac{x}{1} + \frac{6}{x} = \frac{-5}{1}$

1 x 1

1x or x

$\frac{x}{1}\left[\frac{x}{1} + \frac{6}{x} = \frac{-5}{1}\right]$

$x^2 + 6 = -5x$

$\frac{x \cdot x}{1} + \frac{x \cdot 6}{x} = x \cdot -5$

Solve the Following Equations.

3) $\frac{x-7}{x+2} = \frac{1}{4}$ $x \neq -2$

$4(x-7) = 1(x+2)$

$4x - 28 = x + 2$

$-x \quad -x$

$3x - 28 = 2$

$+28 \quad +28$

$\frac{3x}{3} = \frac{30}{3}$

$x = 10 \quad x \neq -2$

4) $\frac{x+5}{2x} + \frac{x+3}{3x} = \frac{1}{3}$

$2 \cdot 3 \cdot x$

$2 \cdot 3 \cdot x \left[\frac{x+5}{2x} + \frac{x+3}{3x} = \frac{1}{3} \right]$

$3(x+5) + 2(x+3) = 2x(1)$

$3x + 15 + 2x + 6 = 2x$

$5x + 21 = 2x$

$-5x \quad -5x$

$\frac{21}{-3} = \frac{-3x}{-3}$

$x = -7$

5) $\frac{3x}{x+5} - \frac{5}{2} = \frac{5}{x+5}$ $2(x+5)$

$x+5$ | 2

$2(x+5)$

$3x(2) - 5(x+5) = 5(2)$

$6x - 5x - 25 = 10$

$x - 25 = 10$

$x = 35$

-36, 9, -4, 5

6) $\frac{6}{x+5} = \frac{x}{6}$

$6 \cdot 6 = (x+5)x$

$36 = x^2 + 5x$

$0 = x^2 + 5x - 36$

$0 = (x+9)(x-4)$

$x = -9$ or 4

7) $\frac{2}{x+5} + \frac{1}{x-5} = \frac{16}{x^2-25}$

Sometimes we will get solutions that are in our list of excluded values.

These solutions are what we call extraneous solutions.

Remember to check every solution to see if it is extraneous.

$t + 3$

$\neq$

$\neq -3$

example: $\frac{1}{n+3} - (n+6) = \frac{n+4}{n+3}$ $\quad n+3$

L.C.D $= n+3$

$\neq -3$

$1 - (n+6)(n+3) = n+4$

$1 - (n^2+3n+6n+18) = n+4$

$1 - (n^2+9n+18) = n+4$

$1 - n^2 - 9n - 18 = n + 4$

$-1 \qquad\qquad\qquad -1$

$-n^2 - 9n - 18 = n + 3$

$+18 \qquad\qquad +18$

$-n^2 - 9n = n + 21$

$+9n \qquad +9n$

$-n^2 = 10n + 21$

$+n^2 \qquad +n^2$

$0 = n^2 + 10n + 21$

$0 = (n+7)(n+3)$

$n = -7$ or -3

21

7 3

10

Homework:

Solving Rational Equations